

The Mystery of the Printing Press: Self-fulfilling debt crises and monetary sovereignty*

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Abstract

Does monetary sovereignty reduce the need for default conditional on weak fundamentals and/or shield government debt markets from self-fulfilling speculative runs? Building on Calvo (1988), we specify a stochastic monetary economy where discretionary policymakers can default on debt holders through surprise inflation or by imposing discrete haircuts, at the cost of both output and budgetary losses. We show that the resort to the printing press per se rules out neither fundamental outright default nor confidence crisis. What matters is the ability of the central bank to issue (interest-bearing) nominal assets (i.e. bank reserves) on which, different from government debt, agents expect no haircut. The scope for successful central bank purchases of government debt is however not unconstrained. We characterize conditions that must be met for this strategy to be credible, i.e. feasible and welfare improving.

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“[...] The proposition [is] that countries without a printing press are subject to self-fulfilling crises in a way that nations that still have a currency of their own are not. The point is that fears of default, by driving up interest costs, can themselves trigger default — and that because there’s a crossing-the-Rubicon aspect to default, once a country crosses that line it will probably impose fairly severe losses on creditors. A country with its own currency isn’t in the same position: even if it is pushed into some inflation, there’s no red line that need be crossed. That’s why America isn’t Greece.

Paul Krugman, “The Printing Press Mystery”, The conscience of a liberal, August 17, 2011, 11:31 AM.

1 Introduction

By the summer of 2012, countries retaining national sovereignty over monetary policy appear to weather fiscal difficulties better than countries without it. Interest rates in countries such as the US and the United Kingdom are low and relative stable, as opposed to the high and variable rates paid by the governments of, say, Ireland, Italy, Portugal and Spain.

Many observers emphasize the ability to honor public liabilities by printing money as one of the key benefits of monetary sovereignty. As synthesized by the Krugman’s quote above, the claim is that the possibility of monetary financing eliminates bad equilibria driven by self-fulfilling expectations, hence shelters countries from the most damaging form of speculation.

While from the historical records it is difficult to separate self-fulfilling and fundamental default, the evidence recently produced by Reinhart and Rogoff (2011) suggests that outright default on domestic debt is actually far from rare, even when governments are in principle able to resort to the printing press. In a long sample ending in 2005, these authors document 68 cases of overt domestic default, against 250 cases of external debt defaults. However, according to the data, domestic default (usually but not necessarily in conjunction with default on external debt) tends to occur under extreme macroeconomic duress — in terms of high inflation and negative growth.¹

The issue is obviously of immediate importance. An effective response to the ongoing sovereign debt crisis requires a clear vision of the interactions between monetary, fiscal and financial policy under conditions of fiscal stress, both in the eurozone, where the problem is most urgent, and elsewhere, where despite the current conditions of relative financial stability the fiscal outlook is not substantially improving.

¹Reinhart and Rogoff (2011) shows that, in the year in which a crisis erupts, on average, output declines by 4 percent if the country defaults on domestic debt, against a decline of 1.2 percent, if the country defaults on external debt only. The corresponding average inflation rates are 170 percent (in cases of domestic debt default) against 33 percent (external debt default).

In this paper, we specify a model in which debt crises may be driven by either self-fulfilling expectations, or weak fundamentals, and explore conditions under which monetary instruments can rule out the former. Building on Calvo (1988), we analyze a two-period economy, where a discretionary government can choose to repudiate, if only partially, its debt by imposing “haircuts” on debt holders, trading off distortionary taxation and the costs of default. Although highly stylized, the model captures the essential “crossing-the-Rubicon” aspect of sovereign default. Providing a generalization of Calvo (1988) to an environment with fundamental fiscal stress and both proportional and fixed costs of default, we first lay down the main mechanism by which multiple equilibria emerge under lack of fiscal commitment.² Due to the assumption of deadweight output costs, self-fulfilling debt crises can occur with sound fundamentals, only for a sufficiently high level of debt.

We then establish the first of our main results: we show that the same mechanism characterizes an economy with monetary sovereignty, that is, where the central bank retains control over monetary policy (i.e. over the inflation rate and seigniorage), *and* the government issues debt denominated in domestic currency. In this environment, in addition to outright haircuts, debt repudiation can obviously result from ex-post surprise inflation. Under discretion, benevolent policymakers trade off the benefits from reducing distortionary taxation, with the macroeconomic costs of inflation. While there is no multiplicity in monetary financing (of the kind discussed by Calvo 1988), the country remains vulnerable to runs on debt, causing the fiscal authorities to opt for outright default even when fundamentals are relatively solid. The main lesson is that the ability to print money is not sufficient to rule out self-fulfilling debt crises. Nor does it necessarily raise the level of debt that can be sustained without incurring in debt runs, depending on the relative magnitude of inflation costs and benefits (in terms of reduced tax distortions).³

As emphasized by Calvo (1988), there is a straightforward policy that can improve welfare by stemming debt runs. An institution could set a ceiling on the interest rate on government debt, as a coordination device shifting market expectations on the fundamental equilibrium.⁴ However, for such a policy to be effective, the announcement has to be fully credible, since doubts about its implementation would undermine any effect on market expectations. Indeed,

²See also Cohen and Villemort (2011) and Cole and Kehoe (2000). Different from Calvo, we model a stochastic economy where default can occur for fundamental reasons, and debt repudiation entails both fixed (output) and variable (budget) costs.

³See also Aguiar et al. (2012). Our results are also relevant in relation to a key conclusion by Calvo (1988). In the monetary economy studied by this author, outright default is ruled out by assumption: multiple equilibria then obtain only by virtue of non-standard costs of inflation. That is, the equilibrium would be unique in the presence of standard convex costs. In a more general specification (such as ours), instead, multiplicity would still be possible, in the rates of outright default.

⁴Such a ceiling on interest rates should be sufficiently low as to rule out the bad equilibrium driven by self-fulfilling crises; as well as high enough to avoid ex ante losses. Namely, too low an interest rate would de facto translate into a transfer of resources covering the short-fall of fiscal revenues under weak fundamentals, effectively amounting to a bailout. As is well understood, anticipations of such a bailout would give rise to moral hazard.

a government lacking commitment cannot pursue this strategy on its own: if investors believe there will be default, they will simply refuse to buy debt at a price inconsistent with their expectations.⁵

Our second set of results concerns the question of whether a central bank can assume this role under any circumstances. This question is intriguing because, from an aggregate perspective, purchases of government debt by the monetary authority are at best backed by its consolidated budget with the fiscal authorities — i.e., there are no additional resources thrown into the game, to augment tax and seigniorage revenues. Nonetheless, central banks do appear to differ from governments in at least two important, and arguably interrelated, respects. The first is the ability of central banks to commit future policies. By way of example, while keeping inflation low in the fundamental equilibrium, the central bank may be able to stem self-fulfilling runs by committing to raise inflation and seigniorage if investors attempt to coordinate away from such equilibrium. We show that monetary authorities can credibly pursue such policy, however, only under the strict condition of a high seigniorage revenue relative to debt.

The second difference concerns the ability of central banks to issue nominal liabilities free from the risk of outright, discrete haircuts, thus exposed only to the risk of inflation — as opposed to government debt, subject to default via both outright haircuts and inflation.⁶ While this ability is obviously not unconstrained, and raises key theoretical and empirical questions, we argue that it plays a key role in determining the success of central bank interventions in the government debt market.⁷

In the last part of the paper, we assume that the central bank can buy government bonds by issuing “reserves” (rather than fiat money), remunerated at a default-free market rate, with no consequences on the value of the initial public debt (to be interpreted as the total financial need of the government after all revenue decisions are taken). While our assumption is consistent with the idea that a monetary backstop to the government does not need to have immediate inflationary consequences (for instance because interventions are “sterilized”), the market rate on the bank liabilities will of course be increasing in expected inflation: the larger the anticipated monetization of debt in the future, the higher this rate. But the purchase of government debt in exchange for reserves raises future inflation only insofar as the primary surplus servicing the debt in the hand of the central bank will fall short of the interest bill on reserves, at the desired level of inflation (and thus seigniorage).

We then analyze two widely-debated views regarding the best strategy for a central bank to intervene in the sovereign debt market. According to one view, monetary authorities should stand ready to buy up to the entire stock of debt

⁵A natural candidate is instead a deep-pocket external lender, such as the IMF, particularly if debt is real (or denominated in foreign currency) – see e.g. Corsetti, Guimaraes, Roubini (2005), Morris and Shin (2006) and Zwart (2007).

⁶In modern economies, high powered money include cash and especially bank reserves, often interest-bearing.

⁷See Gertler and Karadi (2012) for a similar idea applied to unconventional monetary policy.

in the market, relying on an ‘off-equilibrium threat’ to coordinate expectations on the fundamental equilibrium. As is well understood, when successful, such strategy requires no actual intervention. According to a second view, rather than relying on the off-equilibrium threat, the central bank should actually engage in fine-tuned debt purchases, with the goal of steering market interest rates towards their fundamental values. In this case, however, the central bank intervention in reaction to runs would result in a new equilibrium, different from the fundamental one.

We show that, with benevolent policy makers coordinating under a consolidated budget constraint, both strategies are effective in shielding the economy from self-fulfilling runs. Yet, they are not equivalent, in view of their potentially different implications on the incentives for fiscal authorities to act opportunistically. We thus use the model to analyze conditions under which either intervention strategy is credible and thus effective, under the constraint that the central bank always breaks even — that is, it is held responsible for servicing its liabilities in full without relying on fiscal transfer. We find that the second strategy, of fine-tuned interventions in the debt market, retaining market discipline on the fiscal authorities, is more likely to be successful.

The ability to prevent self-fulfilling crises does not rule out sovereign default altogether. Ex-post, defaults may be driven by weak fundamentals and typically associated with debt monetization and inflation — see the evidence in Reinhart and Rogoff (2009) and (2011) discussed above. As weak fundamentals and self-fulfilling expectations may interact in driving the dynamics of actual crises, central bank interventions are inherently exposed to sovereign risk. To be effective they thus require backing by fiscal authorities, recognizing the need to provide support in case of fundamental stress.

While our analytical framework is close to Calvo (1988), our results also build on a vast and consolidated literature on self-fulfilling debt crises, most notably Cole and Kehoe (2002) and more recently Roch and Uhlig (2011), as well as sovereign default and sovereign risk, see e.g. Arellano (2008) and Uribe (2006) among others. A few recent papers and ours complement each other in the analysis of sovereign default and monetary policy. Jeanne (2011) addresses issues in debt runs and lending of last resort using a finite-horizon model with total repudiation, while Aguiar, Amador, Farhi and Gopinath (2012) study a similar problem as in section 2 and 3 below, in a continuous-time framework. Cooper (2012) and Tirole (2012) analyze debt guarantees and international bailouts in a currency union.

By the same token, while we encompass trade-offs across different distortions in a reduced-form fashion, in doing so we draw on a vast literature that has provided micro-foundations, ranging from the analysis of the macroeconomic cost of inflation, in the Kydland-Prescott but especially in the new-Keynesian tradition (see e.g. Woodford 2003), to the analysis of the trade-offs inherent in inflationary financing (e.g. Barro 1983), or the role of debt in shaping discretionary monetary and fiscal policy (e.g. Diaz et al. 2008 and Martin 2009), and, last but not least, the commitment versus discretion debate (e.g. Persson and Tabellini 1993).

The text is organized as follows. Section 2 revisits the logic of self-fulfilling runs on sovereign debt. Section 3 shows that the same mechanism survives under monetary sovereignty, when debt in national currency can also be inflated away. Section 4 discussed the preconditions for interventions in the debt market to stem self-fulfilling debt crises. Section 6 carries out a comparative analysis of backstop policies that can be pursued by monetary authorities. Section 6 concludes, with a brief discussion of lessons for a currency union.

2 The logic of self-fulfilling debt crises

As in Calvo (1988), our main question concerns the determinants of the market price at which a government can borrow a given amount B from private investors at a point in time, and the consequences of agents expectations determining this price on the ex-post fiscal choices by the government.⁸ The model is indeed solved under the maintained assumption that the government is unable to commit credibly to a fiscal plan, detailing how it will service public debt and finance public spending in future periods, under different contingencies. Ex post, it may choose to default, partially or fully, on its liabilities.⁹

Since we are interested in the mechanism by which, for a given level of debt, default is precipitated by agents expectations (rather than, say, in the determinants of public debt accumulation), we model an economy existing for two periods only. In the first period, private agents can invest a given stock of financial wealth W either in domestic public debt B , at the gross market interest rate \tilde{R} , or in a real asset K , with an infinitely elastic supply, yielding an exogenously given “safe” interest rate R . Consumers’ wealth in the first period is thus equal to the both assets, $W = B + K$.

In the second period, the output process is realized. The government sets taxes and may impose a haircut on the owners of government debt at the rate $\theta \in [0, 1]$, inducing distortions that affect net output and aggravate the budget — to be discussed below. All agents are risk neutral: domestic agents derive utility from consuming in period 2 only.

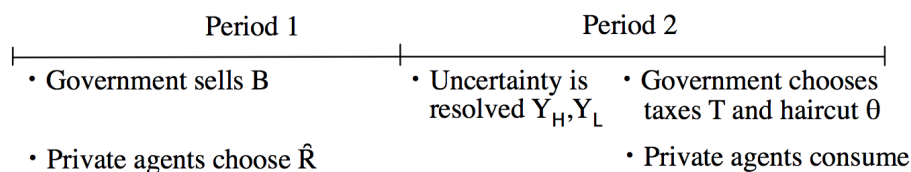
Different from Calvo (1988), we explicitly allow for the possibility that default be driven by fundamental imbalances, in addition to self-fulfilling expectations. To this end, we posit that output varies across two states of the world, H and L , occurring with probability μ and $(1 - \mu)$. As discussed below, assuming two states of the world together with some restrictions on fiscal variables, we will be able to contrast, if only in a stylized way, ‘normal’ and ‘fiscal and macroeconomic stress’ circumstances.

⁸In our two-period economy, the financial need of the government in period 1 coincides with the stock of public debt. In multiperiod models of liquidity runs, there would be a fundamental distinction between the stock of debt B , on which the government may impose haircuts, and the short-term financial needs of the public sector, which determine the exposure of the government to a run — including the primary deficit, interest payments, as well as the rollover of outstanding bonds and bills coming to maturity during the period.

⁹In the model, under commitment there are no self-fulfilling default crises, — as shown by Calvo 1988. For an analysis of default under commitment, see Adam and Grill (2011).

The timeline is summarized by Figure 1 below, emphasizing that fiscal policy cannot be pre-committed and is decided after agents have formed their expectations.

Timeline: Real Model



To clarify the mechanism that may create instability in the debt market, we initially abstract from the monetary dimension of the analysis altogether. A demand for money and the central bank will be introduced from the next section on.

2.1 The optimal choice between taxes and haircuts under discretion

We start with the government choices of the level of taxation and default in period 2 under discretion, i.e. taking the interest rate set by the market in the previous period as given. The policy trade-offs faced in this choice are of course rooted in the distortions that different policy options — defaulting versus running large primary surplus — entail. In the spirit of Calvo, we proceed by specifying the relevant distortions in reduced-form, referring to the relevant literature which has provided micro-foundations to them.

Following the disruption of domestic financial intermediaries and financial markets, sovereign default may entail different types of costs. These include both output and tax losses associated with a contraction of economic activity, as well as transaction costs in the repudiation of government liabilities. In the theoretical literature, some contributions (see e.g. Arellano 2008 and Cole and Kehoe 2000) posit that a default causes output to contract by a fixed amount; in other contributions (see e.g. Calvo 1988) the cost of default falls on the budget and is commensurate to the extent of the haircut imposed on investors. The relative weight of different costs is ultimately an empirical matter — see e.g. Cruces and Trebesch (2012). Yet, alternative assumptions on whether

default costs are mainly lump-sum or proportional to the size of the haircut are bound to shape distinct policy trade-offs with far-reaching implications for policy analysis. For this reason, we find it appropriate not to restrict our model to one type of costs only.

Rather, we follow the literature in assuming, first, that default in period 2 entails a loss of ξ_θ units of output, regardless of the size of default and the state of the economy. This assumption squares well with the presumption that the decision to breach government contracts, even with a small haircut, marks a discontinuity in the effects of such policy on economic activity.¹⁰ Second, we model variable costs of default falling on the budget, proportional to the size of the ex-post haircut $\theta B\tilde{R}$, which of course can be expected to vary across states of the economy.¹¹ We posit that, upon defaulting, the government incurs a financial outlay equal to a fraction $\alpha \in (0, 1)$ of the size of default.¹²

Running a large primary surplus is also distortionary. Namely, in light of the literature on tax smoothing, we posit that taxation results in a dead-weight loss of output indexed by $z(T_i, Y_i)$. Given the level of gross output Y_i , the function $z(\cdot)$ is an increasing and convex function of T , satisfying standard regularity conditions. We realistically assume that, to raise a given level of tax revenue T , output losses are larger, and grow faster in T , if the economy is in a recessionary state, that is:

$$\begin{aligned} z(T, Y_L) &> z(T, Y_H), \\ z'(T, Y_L) &> z'(T, Y_H). \end{aligned}$$

Conversely, for simplicity we posit that government spending G is state invariant — an assumption that is not consequential for our main results.

Under these assumptions, in each state of nature (H or L) the budget constraint of the government in period 2 reads

$$T_i - G = (1 - \theta_i) B\tilde{R} + \alpha\theta_i B\tilde{R}, \quad \alpha, \theta_i \in [0, 1] \quad i = L, H \quad (1)$$

where \tilde{R} is the market interest rate on public debt, set in period 1. The primary surplus — defined as the difference between taxes T and government spending G — finances debt repayment net of the haircut $\theta_i B\tilde{R}$, but gross of the transaction costs of defaulting ($\alpha\theta_i B\tilde{R}$).¹³

¹⁰A plausible alternative assumption could have the fixed costs paid only at a minimum threshold default rate.

¹¹If $\theta_i = 0$, there is no default: the country repays the entire interest bill $B\tilde{R}$ at market rates. If default occurs, repayment is reduced by θ_i .

¹²Calvo (1988) refers to legal and transaction fees associated to default. In a broader sense, one could include disruption of financial intermediaries (banks and pension funds) that may require government support. Note that our results would go through if the variable costs of default were in output, rather than in the budget.

¹³From an accounting perspective, the budget costs of default due to legal fees should be part of the the primary surplus. In what follows, we find it expositively convenient to consider them as part of the debt service, hence we include them in the net interest bill of the government.

In period 2, the budget constraint of the country's residents is

$$C_i = [Y_i - z(T_i, Y_i) - \xi_\theta] - T_i + KR + (1 - \theta_i) B\tilde{R} \quad (2)$$

Consumption is equal to output, Y , net of output losses from raising taxes and defaulting on liabilities, $z(T_i, Y_i) + \xi_\theta$, minus the tax bill, T , plus the revenue from portfolio investment. Consumers earn the safe (gross) interest rate R on their holdings of K , and the net (ex haircut) payoffs $(1 - \theta)\tilde{R}_i$ on their holding of public debt B .

Under discretion, the government decides its optimal policy plan (θ_i, T_i) by maximizing agents' utility (which coincides with consumption C_i), subject to its budget constraint and taking expectations (and thus \tilde{R}) as given. The optimal discretionary plan is characterized by two notable features. First, fixed costs of default ξ_θ induce a non-linearity: a positive θ_i will be chosen if and only if the benefits of the haircut will be large enough compared to this cost. Second, there is a well-defined upper bound on the country's willingness to raise taxes, which vary depending on whether the country chooses to default or service its liabilities in full.

Conditional on default, let \hat{T}_i denote the level of taxes that maximizes private consumption under policy discretion (taking \tilde{R} as given). As long as an interior solution for θ_i exists, i.e., the constraint $\theta_i \leq 1$ is not binding, the first order condition of the policy problem yields¹⁴

$$z'(\hat{T}_i, Y_i) = \frac{\alpha}{1 - \alpha} \quad (3)$$

Conditional on default, the government chooses an optimal tax level \hat{T}_i trading-off the economic costs associated with raising revenue $z(T_i)$, with the variable budget cost of default, indexed by the parameter α . This trade-off is independent of spending and the interest rate. Note that the optimal taxation level \hat{T}_i determines the maximum primary surplus that the country finds it optimal to generate conditional on default, $\hat{T}_i - G$, in turn nailing down net output $Y_i - z(\hat{T}_i)$ as well as the optimal haircut rate. It may of course happen that the constraint $\theta_i \leq 1$ is binding in equilibrium. In this case, the government sets a tax level higher than \hat{T}_i , to cover current non-interest expenditure including the variable budget costs of default evaluated at $\theta_i = 1$, namely:

$$\hat{T}_i \leq \bar{T}_i = G + \alpha B\tilde{R} \quad (4)$$

Conditional on the government choosing not to default, $\theta_i = 0$, tax revenue needs to rise enough to finance both current spending G and the debt service

¹⁴This is just the first order condition from choosing θ_i to maximize ex-post consumption C_i subject to 1:

$$\begin{aligned} -z'(T_i, Y_i) \frac{\partial T_i}{\partial \theta_i} - \frac{\partial T_i}{\partial \theta_i} - B\tilde{R} &= 0, \\ \frac{\partial T_i}{\partial \theta_i} &= -(1 - \alpha) B\tilde{R} \end{aligned}$$

$B\tilde{R}$ in full:

$$\tilde{T}_i - G = B\tilde{R} = \tilde{T} - G \quad (5)$$

Note that with a state invariant G , the primary surplus required to service the outstanding liabilities is the same across states.

How far is the government willing to raise taxes before exercising (optimally) the option to default? The ‘fiscal capacity’ of the government is naturally defined as the maximum primary surplus that the country will find it optimal to generate to finance its interest bill in full. Comparing consumption under full debt service and default identifies such threshold primary surplus, that can be written as the following function of the fixed cost ξ_θ :¹⁵

$$\tilde{T}_i \leq \hat{T}_i + \xi_\theta + \frac{B\tilde{R} + G - \hat{T}_i}{1 - \alpha} - \left[z(\tilde{T}_i, Y_i) - z(\hat{T}_i, Y_i) \right] \quad (6)$$

Relative to the optimal taxation conditional on default \hat{T}_i , the fiscal capacity of a country is pinned down by the fixed output costs ξ_θ plus the variable budget costs of default $\frac{B\tilde{R} + G - \hat{T}_i}{1 - \alpha}$ (the latter expressed in terms of forgone income), minus the incremental output loss due to tax distortions when the debt is repaid in full — the term in squared bracket. The larger this term, or the lower the overall default costs $\xi_\theta + \frac{B\tilde{R} + G - \hat{T}_i}{1 - \alpha}$, the lower the fiscal capacity of the country.

Ultimately, the fiscal capacity is a function of the budget cost of default and the level of debt. Since \hat{T}_i is increasing in α , a high budget cost of default raises fiscal capacity. Conversely, a high stock of debt reduces it, via its effect on \tilde{T}_i . To see this point most clearly, we rewrite the above condition as follows:

$$\xi_\theta \geq z(G + B\tilde{R}, Y_i) - z(\hat{T}_i(\alpha), Y_i) - \frac{\alpha}{1 - \alpha} \left[B\tilde{R} - (\hat{T}_i(\alpha) - G) \right]$$

where for simplicity we have assumed that the constraint $\theta \leq 1$ is not binding in equilibrium. It should be clear by now that the term ‘capacity’ by no means indicates a technical limit, but is the outcome of a discretionary, welfare-maximizing decision by the government.

The above conditions are defined up to the size of the haircuts, to be determined jointly with equilibrium pricing by private markets.

¹⁵Namely:

$$\begin{aligned} \tilde{T}_i &\leq - \left[z(\tilde{T}_i, Y_i) - z(\hat{T}_i, Y_i) \right] + \hat{T}_i + \xi_\theta + \theta_i B\tilde{R} \\ \tilde{T}_i - G &= B\tilde{R} \\ \hat{T}_i - G &= [1 - \theta_i(1 - \alpha)] B\tilde{R} \\ &=> \\ \tilde{T}_i &\leq \xi_\theta - \left[z(\tilde{T}_i, Y_i) - z(\hat{T}_i, Y_i) \right] + \hat{T}_i + \frac{B\tilde{R} + G - \hat{T}_i}{1 - \alpha} \end{aligned}$$

2.2 Debt pricing and equilibrium restrictions

The price of debt is pinned down by the interest parity condition, equating (under risk neutrality) the expected real returns on domestic bonds to the safe interest rate:

$$\tilde{R}[\mu(1 - \theta_H) + (1 - \mu)(1 - \theta_L)] = R \quad (7)$$

Under rational expectations, agents anticipate the optimal discretionary plan of the government conditional on the market interest rate \tilde{R} .

This condition, together with the conditionally optimal tax rates (3) or (4) and (5), the condition for choosing default (6), and the government budget constrain (1), define an equilibrium.

As already mentioned, we want to encompass in our model the possibility of fundamental fiscal stress. For this reason, in addition to assuming that output varies stochastically between a high and a low level, we impose two sets of conditions ensuring the existence of equilibria with the desired properties. First we assume B to be large enough that, in the low output state, the primary surplus under default will fall short of the interest bill of the government valued at the risk-free rate R :

$$\hat{T}_L - G < BR. \quad (8)$$

This implies that, unless the fixed cost ξ_θ is prohibitively high, the government will default for fundamental reasons in the low-output state. Conversely, we posit that, given B , the primary surplus optimally chosen under default in the high-output state can comfortably finance the largest possible interest bill — corresponding to the case in which agents anticipate total repudiation in the low-output state:¹⁶

$$\hat{T}_H - G > \frac{R}{\mu}B. \quad (9)$$

So, there is no fundamental reason for defaulting in the high-output state.

Second, we further restrict B and parameters such that, when agents anticipate complete default in the low-output state and no default in the high one, the optimal primary surplus (including the variable budget costs of defaulting) in L is non-negative,

$$\hat{T}_L - G - \alpha \frac{R}{\mu}B \geq 0. \quad (10)$$

Hence, the haircut rate in this state is less than 100 percent. Note that, together with (8), the above condition restricts μ (the probability of the good output state) to be higher than α (the proportional budget cost of default). By the same token, we posit

$$BR > \frac{(\alpha + \mu)}{(1 - \alpha)} (\hat{T}_H - \hat{T}_L) \quad (11)$$

¹⁶Note that a countercyclical G would increase ‘fiscal stress’ in the low output state, while raising fiscal surplus in the good output state. Generalizing our model in this direction would aggravate notation, without producing additional insight.

which is a sufficient condition for a default to occur (per effect of self-validating expectations) also in the high output state, with an haircut rate less than 100 percent.

The overall function of these assumptions is straightforward: they ensure that (a) agents price the possibility that the government chooses to default on its liabilities, for purely fundamental reasons, and that (b), in response to a speculative run, default occurs in both the low and the high-output state.¹⁷

2.3 Weak fundamentals and self-validating expectations as drivers of sovereign debt crises

We will now show that, depending on the relative weight of the costs of default and taxation, and the level of debt, different equilibrium outcomes are possible in the model, and the equilibrium is not necessarily unique. For expositional reasons, it is convenient to proceed in two steps. In a first step, we extend the main result by Calvo (1988) — who posits no fixed costs of default — to our stochastic setting. We show that, if $\xi_\theta = 0$, there will be two equilibria. One is a fundamental equilibrium (denoted with the superscript F), in which the interest rate charged on debt reflects anticipations of default in the low-output state of nature in period 2, based on the correct probability that this state materializes. The other one is a non-fundamental equilibrium (denoted with N), in which market participants coordinate their expectations on default occurring in both the high and low output state — and thus charge a higher equilibrium interest rate than in F . This result is summarized by the following proposition:

Proposition 1 *In the economy summarized by (1), (2), and (7), where we posit $\xi_\theta = 0$, with the government optimally choosing taxes satisfying either (3) or (4) in case of default, depending on whether the constraint $\theta_i \leq 1$ is/is not binding, and (5) otherwise, under the maintained assumptions (8), (9), (10) and (11), namely, if $\mu > \alpha$ and the following restrictions on the initial debt level hold:*

$$\begin{aligned} \mu \left(\widehat{T}_H - G \right) &> BR > \text{Max} \left\{ \widehat{T}_L - G, \frac{(\alpha + \mu)}{(1 - \alpha)} \left(\widehat{T}_H - \widehat{T}_L \right) \right\} \\ \widehat{T}_L - G &\geq \frac{(\alpha + \mu)}{(1 - \alpha)} \left(\widehat{T}_H - \widehat{T}_L \right) \iff (1 + \mu) \left(\widehat{T}_L - G \right) \geq (\alpha + \mu) \left(\widehat{T}_H - G \right) \end{aligned}$$

an equilibrium will exist and will not be unique. There will be a fundamental equilibrium in which default will occur only in the low output state of the world, with the equilibrium haircuts given by $\theta_H^F = 0$ and

$$0 < \theta_L^F = \frac{RB - \left(\widehat{T}_L - G \right)}{(1 - \mu) \left[RB - \left(\widehat{T}_L - G \right) \right] + (\mu - \alpha) RB} < 1. \quad (12)$$

¹⁷As in Calvo (1988), the initial stock of debt is not so high that there is no equilibrium price at which it can be sold to market participants.

There will be another equilibrium, driven by self-validating expectations, where $\theta_L^F < \theta_L^N$ and $0 < \theta_H^N \leq \theta_L^N$, with the rate of default in each state given by:

$$\begin{aligned}\theta_H^N &= \frac{\left(\widehat{T}_H - G - BR\right) - \frac{(1-\mu)}{(1-\alpha)}\left(\widehat{T}_H - \widehat{T}_L\right)}{(1-\alpha)\left(\widehat{T}_H - G - BR\right) - (1-\mu)\left(\widehat{T}_H - \widehat{T}_L\right) + \alpha\left(\widehat{T}_H - G\right)} < \text{(13)} \\ \theta_L^N &= \min \left\{ \frac{\left(BR - \widehat{T}_L + G\right) - \frac{\mu}{(1-\alpha)}\left(\widehat{T}_H - \widehat{T}_L\right)}{(1-\alpha)\left(BR - \widehat{T}_L + G\right) - \mu\left(\widehat{T}_H - \widehat{T}_L\right) + \alpha\left(\widehat{T}_L - G\right)}, 1 \right\}.\end{aligned}$$

Proof. See appendix. ■

The equilibrium interest rate will generally be higher than the safe rate R . In the F-equilibrium, the difference is determined by expectations of default in the weak state. In the N-equilibrium, the difference is driven by self-confirming beliefs that the fiscal authority will default regardless of the level of output.¹⁸ Note that, by virtue of the conditions stated at the end of the previous subsection, the haircut in the low-output state is strictly below 100 percent in the F-equilibrium. In the N-equilibrium, in contrast, no condition prevents self-validating expectations from pushing the government in this contingency to default on its entire stock of debt.

Haircuts and interest rates vary across equilibria. In the fundamental equilibrium, if the government defaults, it does so only in the low output state Y_L ; in the non-fundamental equilibrium, the government imposes haircuts in both states of the world.

The logic of multiplicity is illustrated graphically below. Two graphs plot, against the market rate \widetilde{R} , the best-response default rate θ that satisfies the budget constraint and the optimality conditions of the government (light-colored line) and of the investors (dark-colored line), in the high-output and the low-output state, respectively. In each state of the world, the government best-response depends on the haircut in the other states only through the market interest rate \widetilde{R}

$$\theta_{i\text{-Government}} = \frac{B\widetilde{R} + G_i - \widehat{T}_i}{(1-\alpha)B\widetilde{R}} \quad i = L, H$$

Conversely, from (7), the state-contingent haircut expected by investors depends not only on \widetilde{R} , but also the expected haircut in the other states.

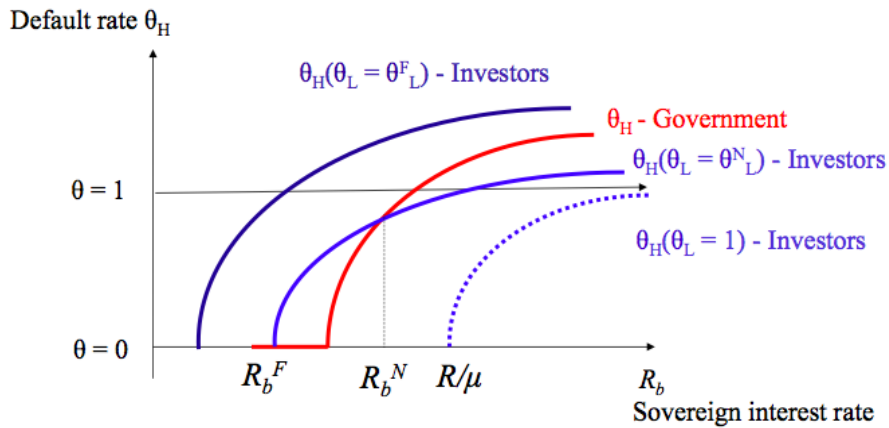
Focusing first on the figure for the high output state, the investors best response to \widetilde{R} , i.e.,

$$\theta_{H\text{-Investors}} = 1 - \frac{1}{\mu} \left[\frac{R}{\widetilde{R}} - (1-\mu)(1-\theta_L) \right]$$

¹⁸The solution in Calvo (1988) is a special case of our analysis if, in addition to assuming $\xi_\theta \rightarrow 0$ (no fixed cost of default), we let $\mu \rightarrow 1$ (output is non stochastic). In the non-stochastic version of the model, the equilibrium may be unique for a special combinations of parameters' values.

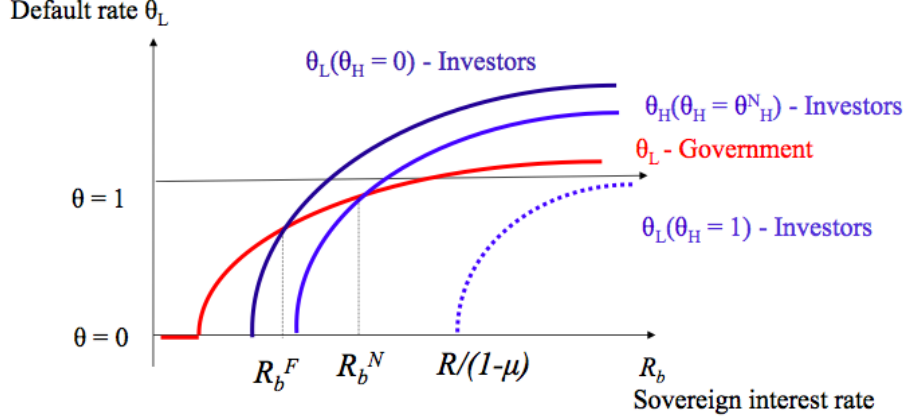
is plotted under the assumptions that, in the low-output state, there is, alternatively: fundamental default (the curve to the left, with $\theta_L = \theta_L^F$), non-fundamental default (the curve in the center, with $\theta_L = \theta_L^N$), complete default (the dotted curve to the right, with $\theta_L = 1$). Of these three curves, only one (conditional on $\theta_L = \theta_L^N$) crosses the government best response at a positive haircut rate: a default in the high-output state can only occur conditional on investors coordinating on self-validating expectations of fiscal stress also in the low-output state.

State of the economy with sound fundamentals (H)



The next figure, drawn for the low-output state, depicts quite a different situation. Here, the government best response crosses *two* best responses for the investors, one conditional on no default in the high-output state (the fundamental equilibrium, with $\theta_H = 0$); the other conditional on default in this state (the non-fundamental equilibrium, with $\theta_H = \theta_H^N$).

State of the economy with weak fundamentals (L)



Having revisited Calvo (1988), we now take a step further, and derive the implications of fixed output costs of default, typically assumed by the literature on sovereign risk. We will show that, first, these costs identify a threshold value for debt, below which there is no multiplicity. Second, in the range of debt where equilibrium is unique, default may not occur at all, not even in the low-output state. These results are formally stated in the following proposition.

Proposition 2 *In the same economy described in proposition 1, for given fixed output costs of default ($\xi_\theta > 0$):*

(a) *If the government debt B is sufficiently low so that (6) holds in the high-output state, namely B satisfies the following inequality:*

$$\xi_\theta \geq z(G + B\tilde{R}^N, Y_H) - z(\hat{T}_H, Y_H) - \frac{\alpha}{1-\alpha} \left[B\tilde{R}^N - (\hat{T}_H - G) \right] \quad (14)$$

where

$$\begin{aligned} B\tilde{R}^N &= \frac{RB}{1 - \mu\theta_H^N - (1-\mu)\theta_L^N} = \\ &= \frac{(1-\mu) \left[RB - (\hat{T}_L - G) \right] + (\mu - \alpha) RB - \mu (\hat{T}_H - G)}{\alpha} \end{aligned}$$

there is a unique, fundamental equilibrium.

In this unique equilibrium, default will occur in the low output state only, provided (6) does not hold in this state, namely for a level of debt B satisfying the following inequality:

$$\xi_\theta < z(G + B\tilde{R}^F, Y_L) - z(\hat{T}_L, Y_L) - \frac{\alpha}{1-\alpha} \left[B\tilde{R}^F - (\hat{T}_L - G) \right] \quad (15)$$

where

$$B\tilde{R}^F = \frac{RB}{1 - (1 - \mu)\theta_L^F} = \frac{(1 - \mu) \left[RB - (\hat{T}_L - G) \right] + (\mu - \alpha) RB}{(\mu - \alpha)}$$

with the equilibrium rate of default being given by (12).¹⁹ For a lower level of debt, the fundamental equilibrium will display no default: $\theta_H^F = \theta_L^F = 0$.

(b) If the stock of debt is large enough that (6) is violated in the high output state, namely (14) does not hold, the equilibrium will generally be not unique.

There will be two equilibria, characterized as the F- and the N-equilibrium in Proposition 1.

Proof. See appendix. ■

Our second proposition establishes that the equilibrium is unique for levels of debt that are low in relation to the non-variable costs of default. Specifically, a very low level of debt may discourage credit events even when the macroeconomic outcome turns out to produce fiscal stress — default costs would reduce welfare more than the distortions of running high fiscal surpluses in a downturn. Under these circumstances, haircuts become an attractive option only when the legacy debt of the government is sizeable enough. Still, the fixed cost ξ_θ may ensure uniqueness of equilibrium, insofar as they are large enough to rule out default in the high output case.

These different possibilities are illustrated by the figure below, plotting the right hand side of (14) and (15), together with the fixed costs of default, against a given initial stock of debt. The locus in the center of the figure is the relative net (variable) benefits from defaulting in the high output state in the non-fundamental equilibrium. The other locus is the corresponding net benefit from defaulting in the low state, in the fundamental equilibrium. Since the latter locus can lie above or below the former, the figure shows two of the same. It can be shown that, although all these loci may be non-linear over some regions

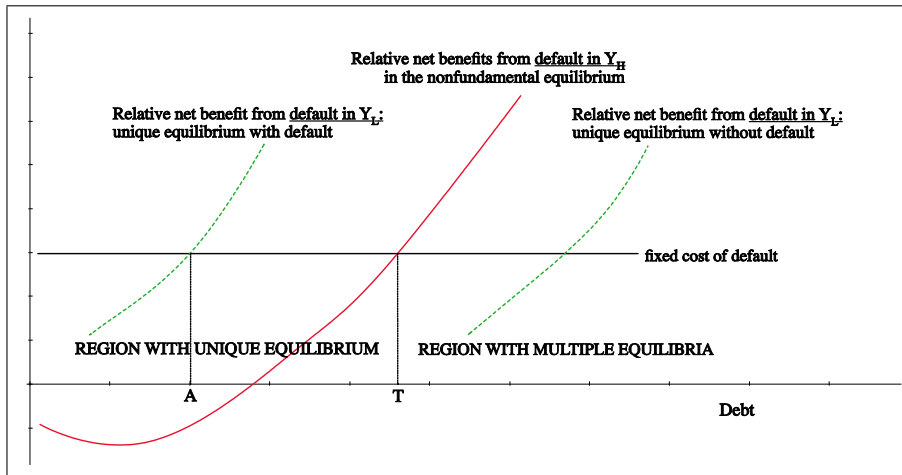
¹⁹Given the assumptions that $z(T, Y_L) > z(T, Y_H)$, $z(\hat{T}_H, Y_H) - z(\hat{T}_L, Y_L) \geq \frac{\alpha}{1 - \alpha} (\hat{T}_H - \hat{T}_L) > 0$, a necessary and sufficient condition for a range of debt to exist so that such an equilibrium with fundamental default is possible, is that for $B \geq \underline{B}$:

$$\begin{aligned} z(\underline{B}\tilde{R}^F + G, Y_L) - z(\hat{T}_L, Y_L) - \frac{\alpha}{1 - \alpha} (\underline{B}\tilde{R}^F + G - \hat{T}_L) &\geq \xi_\theta \\ \underline{B} : z(\underline{B}\tilde{R}^N + G, Y_H) - z(\hat{T}_H, Y_H) - \frac{\alpha}{1 - \alpha} (\underline{B}\tilde{R}^N + G - \hat{T}_H) &= \xi_\theta \end{aligned}$$

$$\begin{aligned} B\tilde{R}^N &= \frac{RB}{1 - \mu\theta_H^N - (1 - \mu)\theta_L^N} = \frac{(1 - \mu) \left[RB - (\hat{T}_L - G) \right] + (\mu - \alpha) RB - \mu (\hat{T}_H - G)}{\alpha} \\ B\tilde{R}^F &= \frac{RB}{1 - (1 - \mu)\theta_L^F} = \frac{(1 - \mu) \left[RB - (\hat{T}_L - G) \right] + (\mu - \alpha) RB}{(\mu - \alpha)}. \end{aligned}$$

of debt, they are always upward sloping around the point at which they cross the fixed-cost horizontal line, as depicted in the figure.

Consider the region of debt to the left of the threshold T below which the equilibrium is unique (determined by the condition (14)). In this region, default in the low-output state may or may not occur. When the locus (15) lies above the horizontal line (14), the government will default in the low output state only if the initial debt is comprised between A and T — debt is sufficiently high to raise the benefits of haircuts above its fixed costs. Conversely, there will be no initial level of debt at which the government will default if the locus (15) happens to lie below (15).



According to our model, the self-fulfilling crises emphasized by Calvo (1988) emerge as a possibility only for a relatively high stock of government liabilities in relation to the fixed costs of default, similarly to Cole and Kehoe (2000). Fixed costs thus may explain while defaults are not frequent at relatively low debt level.

3 Sovereign default in a monetary economy with non-indexed debt

In the previous section, we have analyzed a mechanism that potentially makes a country vulnerable to self-fulfilling sovereign debt crises — the ‘crossing-the-Rubicon aspect of default’ as labelled by Krugman. In this section, we ask whether granting a country monetary sovereignty — that we define as both a printing press *and* the ability to issue debt denominated in domestic currency — would be enough to eliminate multiple equilibria, thus ensuring that ‘there is no red line to be crossed.’

As stressed by Calvo (1988), some degree of repudiation is a natural outcome in a monetary economy, because unexpected changes in inflation rates affect the ex-post real returns on assets which are not indexed to the price level.

Repudiation in period 2 can thus take the form of either outright default on debt holdings, or a reduction in the real value of debt through surprise in ex-post inflation, or both.²⁰

The question we are interested in is whether the options to inflate public debt ex post and raising revenue through the inflation tax dispose of equilibria in which the government ends up resorting to outright default per effect of self-fulfilling expectations. To address this question, we focus on the benchmark policy scenario in which both the fiscal and the monetary authorities, while acting under discretion, are benevolent (maximize the utility of the representative agent) and act under their consolidated budget constraint. Moreover, as in the literature on discretionary policy and default, we stipulate that the (consolidated) budget constraint has to be satisfied for every policy strategy. We also intentionally abstract from issues in the determination of the value of nominal liabilities in the first, initial period, of the kind analyzed by the fiscal theory of the price level and related literature (see e.g. Uribe 2006 for a related approach).

In what follows, we will show that, contrary to the claim by Krugman (or at least to a superficial reading of it) the same non-uniqueness of equilibria analyzed in Section 2 also characterizes the monetary version of our economy where public debt is nominal.

3.1 The model setup

To minimize the use of new notation, we introduce the following modifications to our model specification. First, the initial stock of government liabilities is now defined in nominal, rather than in real terms. Its ex-post value will then vary with realized inflation in period 2. Second, as in Calvo (1983), we restrict our attention to unit-velocity demand for (non-interest bearing) money M :

$$M/P = \kappa, \tag{16}$$

where P is the price level. The seigniorage revenue — the amount of real resources the government can obtain by increasing the stock of high-powered money — in the second period will thus be:

$$\frac{M_i - M_1}{P_i} = \frac{\pi_i}{1 + \pi_i} \kappa, \quad i = L, H \tag{17}$$

where π_i ($\infty > \pi_i > -1$) is the inflation rate between period 1 and 2; as before, variables in the last period are indexed to the random realization of the high- and low-output states of the world. In addition to B , also M_1 and P_1 are exogenously given.²¹

²⁰This is different from the monetary model analyzed by Calvo (1983), where partial repudiation exclusively takes the form of inflation.

²¹It is easy to generalize this framework to the case $\kappa(\pi)$, so to encompass a Laffer curve in the analysis.

In period 2, the budget constraint of the government reads:

$$T_i - G = [1 - \theta_i (1 - \alpha)] \frac{B}{1 + \pi_i} \tilde{R} - \frac{\pi_i}{1 + \pi_i} \kappa, \quad \alpha, \theta_i \in [0, 1] \quad i = L, H \quad (18)$$

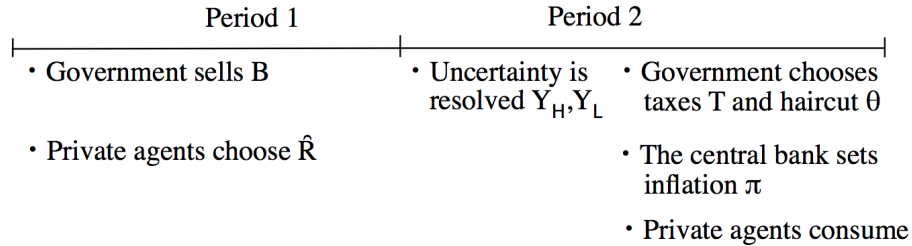
A primary deficit — defined as the difference between government spending G and taxes T — can be financed at least in part through the inflation tax. The consumption/budget constraint of the country residents is

$$C_i = [Y_i - z(T_i, Y_i)] - C(\pi_i) - \xi_\theta - T_i - \frac{\pi_i}{1 + \pi_i} \kappa + KR + (1 - \theta_i) \frac{B}{1 + \pi_i} \tilde{R}, \quad (19)$$

where $C(\cdot)$ is the convex cost of inflation such that $C(0) = C'(0) = 0$ — a standard instance being given by $C(\pi) = \frac{\lambda}{2} \pi^2$. Consumption is equal to output Y_i net of losses from raising taxes and inflation, minus the costs of default (if any), minus the tax bill T_i including the inflation tax $\frac{\pi_i}{1 + \pi_i} \kappa$, plus the revenue from portfolio investment. The net real payoffs on public debt is $(1 - \theta_i) \frac{B}{1 + \pi_i} \tilde{R}$.

The timeline is summarized by the Figure below.

Timeline: Monetary Model



3.2 The optimal discretionary choice of inflation, taxation and default

The optimal policy plan under discretion (taking market expectations and thus \tilde{R} as given) is defined over T_i, θ_i , and π_i . These instruments could be controlled by different policymakers, raising issues in the specification of their objective functions and constraints, and the way they interact strategically. However, for

the purpose of verifying whether the option to monetize the debt and the availability of seigniorage revenue reduces the vulnerability to self-fulfilling sovereign debt crisis, a natural benchmark to focus on is the case in which benevolent (discretionary) fiscal and the monetary authorities set their plans under coordination, hence subject to their consolidated budget constraint. Such benchmark provides a reference allocation, against which to assess potentially interesting alternatives, revolving around political economy considerations or institutional settings, which may differentiate the objectives and constraints of the monetary and fiscal authorities. We should note here that, even when the monetary and fiscal authorities are operationally independent, a common objective function and budget constraint fundamentally narrow the scope for opportunistic behavior. Under discretion, indeed, the policy plan below will be the same under Nash.²²

According to the optimal discretionary plan, inflation and taxes are chosen by trading off the output benefits from reducing the need for distortionary income taxation and the costs of default (if any), with the output cost of inflation, according to the following condition

$$z'(T_i, Y_i) \left(B\tilde{R} + \kappa \right) - \theta_i B\tilde{R} [\alpha - z'(T_i, Y_i) (1 - \alpha)] = (1 + \pi_i)^2 \mathcal{C}'(\pi_i) \quad (20)$$

where the tax level of course depends on whether the government defaults. Observe that the inflation rate would not be equal to zero even if printing money generated no seigniorage revenue ($\kappa = 0$). This is because a discretionary monetary authority will not resist the temptation to inflate the stock nominal debt, if only moderately so (according to the condition above).²³ On the other hand, positive costs of inflation prevent policymakers from wiping away the debt with infinite inflation.

Conditional on default, the optimal upper bound on the country's willingness to raise distortionary taxes is the same as before. If the constraint $\theta_i \leq 1$ is not binding, taxes will satisfy

$$z'(\hat{T}_i, Y_i) = \frac{\alpha}{1 - \alpha}, \quad (21)$$

implying that the optimal inflation rate obeys the following trade-off:

$$\frac{\alpha}{1 - \alpha} \left(B\tilde{R} + \kappa \right) = (1 + \hat{\pi})^2 \mathcal{C}'(\hat{\pi}). \quad (22)$$

Note that, in this case, the optimal inflation rate is the same across states of the

²²Fiscal and monetary policies could also be set sequentially, with one of the authorities acting as the leader — i.e., internalizing the reaction function of the other. It can be shown that, as long as both authorities have the same objective function and constraint, results tend to be either identical (when the fiscal authority leads), or quite close (when the monetary authority leads), to the one discussed in the main text.

²³Under commitment the monetary authority would choose a lower inflation rate. However, it would not be able to undo the multiplicity due to the lack of commitment by the fiscal authority in choosing the size of the haircuts. We discuss commitment later on in the paper.

world, i.e., $\hat{\pi}_H = \hat{\pi}_L = \hat{\pi}$.²⁴ If the constraint $\theta_i \leq 1$ is binding in equilibrium, instead, taxes and seigniorage will have to cover current non-interest expenditure. Specifically, taxes will have to be larger than \hat{T}_i (and state contingent):

$$\hat{T}_i \leq \bar{T}_i = G + \alpha B \frac{\tilde{R}}{1 + \pi_i} - \frac{\pi_i}{1 + \pi_i} \kappa \quad (23)$$

while inflation will be correspondingly set according to

$$z'(\bar{T}_i, Y_i) (B\tilde{R} + \kappa) = (1 + \pi_i)^2 \mathcal{C}'(\pi_i) \quad (24)$$

Conditional on no default ($\theta_i = 0$), the revenue from taxation and seigniorage need to finance the government real expenditure and interest bill in full

$$\tilde{T}_i + \frac{\tilde{\pi}_i}{1 + \tilde{\pi}_i} \kappa - G = \frac{B\tilde{R}}{1 + \tilde{\pi}_i} \quad (25)$$

where the tax and the inflation rates are set according to (20) with $\theta_i = 0$, that is

$$z'(\tilde{T}_i, Y_i) (B\tilde{R} + \kappa) = (1 + \tilde{\pi}_i)^2 \mathcal{C}'(\tilde{\pi}_i). \quad (26)$$

Both \tilde{T}_i and $\tilde{\pi}_i$ are always state-contingent in this case.²⁵

As in the previous section, the ‘fiscal capacity’ of the government is defined as the maximum taxation and seigniorage revenue the government is willing to raise to service its liabilities in full. In our monetary economy, it is determined by the following condition:

$$\begin{aligned} \tilde{T}_i + \frac{\tilde{\pi}_i}{1 + \tilde{\pi}_i} \kappa \leq & \xi_\theta + \hat{T}_i + \frac{\hat{\pi}}{1 + \hat{\pi}} \kappa - \left[z(\tilde{T}, Y_i) - z(\hat{T}_i, Y_i) \right] \\ & - [\mathcal{C}(\tilde{\pi}_i) - \mathcal{C}(\hat{\pi}_i)] + (1 - \alpha)^{-1} \left[G + \frac{\tilde{R}}{1 + \tilde{\pi}_i} B - \hat{T}_i - \frac{\hat{\pi}_i}{1 + \hat{\pi}_i} \kappa \right] \end{aligned}$$

The ‘fiscal capacity’ of a country is now a function of the incremental costs of inflation, when the debt is repaid in full rather than partially.

²⁴This property of the optimal inflation rate depends on the simplifying assumption that the cost of inflation does not vary with the state of the world. It would be easy to relax this assumption, at the cost of cluttering the notation without much gain in terms of economic intuition.

²⁵To see this, rewrite the implicit condition for inflation replacing \tilde{T}_i :

$$z'(B\tilde{R} + G - \frac{\tilde{\pi}_i}{1 + \tilde{\pi}_i} \kappa, Y_i) (B\tilde{R} + \kappa) = (1 + \tilde{\pi}_i)^2 \mathcal{C}'(\tilde{\pi}_i).$$

Since the function $z'(\tilde{T}_i, Y_i)$ is state contingent, also the left-hand-side has to be state contingent.

To highlight the role of the inflation tax, we can rewrite the above condition as follows, again assuming that the constraint $\theta \leq 1$ is not binding:

$$\begin{aligned} \xi_\theta \geq & z\left(G + \frac{\tilde{R}B - \tilde{\pi}_i\kappa}{1 + \tilde{\pi}_i}, Y_i\right) - z(\hat{T}_i, Y_i) + [\mathcal{C}(\tilde{\pi}_i) - \mathcal{C}(\hat{\pi}_i)] \\ & - \frac{\alpha}{1 - \alpha} \left[\frac{\tilde{R}B - \hat{\pi}_i\kappa}{1 + \hat{\pi}_i} - (\hat{T}_i - G) \right] + \left(\frac{1 + \hat{\pi}_i}{1 + \tilde{\pi}_i} - 1 \right) \frac{\tilde{R}}{1 + \hat{\pi}_i} B \end{aligned} \quad (27)$$

The above optimal conditions are defined up to the size of the haircut, to be determined jointly with equilibrium pricing by private markets.

3.3 Debt pricing and equilibrium restrictions

The interest parity condition, pinning down that price of government debt, now includes expected inflation:

$$\tilde{R}[\mu(1 - \theta_H) + (1 - \mu)(1 - \theta_L)] = [\mu(1 + \pi_H) + (1 - \mu)(1 + \pi_L)] R. \quad (28)$$

Under risk neutrality, expected real returns are the same on government bonds and on the real asset.

The rational expectations equilibrium is defined by these pricing conditions, together with the budget constraint (18), the two conditional optimal tax rates, either (21) or (23), or (25), optimal inflation, either (22) or (24), and the condition for choosing default (27).

Below we rewrite the conditions ensuring that our economy is under fundamental fiscal stress in the low output state, but not in the high output state. Similarly to the real economy, we posit that, in the low-output state, the government revenue under fundamental default will fall short of the interest bill of the government valued at the nominal risk-free rate $\left[\mu(1 + \tilde{\pi}_H) + (1 - \mu)(1 + \hat{\pi}^F) \right] R$. In the nominal version of the model, this must be true adding the revenue from the inflation tax, $\frac{\hat{\pi}^F}{1 + \hat{\pi}^F} \kappa$, to that from taxation:

$$\hat{T}_L - G + \frac{\hat{\pi}_L^F}{1 + \hat{\pi}_L^F} \kappa < \left[\mu \frac{1 + \tilde{\pi}_H}{1 + \hat{\pi}_L^F} + (1 - \mu) \right] RB < RB. \quad (29)$$

In the low-output state, unless the fixed cost ξ_θ is prohibitively high, the government will default for fundamental reasons.

Conversely, we assume that, in the high-output state, there will be no fundamental reason for defaulting. The primary surplus net of the inflation tax revenue will be above the largest possible interest bill, when agents anticipate total repudiation in the low-output state:

$$\hat{T}_H^F + \frac{\tilde{\pi}_H}{1 + \tilde{\pi}_H} \kappa - G > \left[\mu \frac{1 + \tilde{\pi}_H}{1 + \hat{\pi}_L^F} + (1 - \mu) \right] \frac{R}{\mu} B \quad (30)$$

We also impose the analogs of (10) and (11): parameters are such that, when agents anticipate complete default in the low-output state and no default in the high one, the primary surplus (including the variable budget costs of defaulting) in L is non-negative

$$\widehat{T}_L + \frac{\widehat{\pi}_L^F}{1 + \widehat{\pi}_L^F} \kappa - G - \left[\mu \frac{1 + \widetilde{\pi}_H}{1 + \widehat{\pi}_L^F} + (1 - \mu) \right] \frac{\alpha}{\mu} RB \geq 0. \quad (31)$$

As above, the above conditions restrict μ (the probability of the good output state) to be higher than α (the proportional budget cost of default). By the same token, we posit

$$\left[\mu \frac{1 + \widetilde{\pi}_H}{1 + \widehat{\pi}_L^F} + (1 - \mu) \right] BR > \frac{(\alpha + \mu)}{(1 - \alpha)} \left(\widehat{T}_H + \frac{\widehat{\pi}_H^F}{1 + \widehat{\pi}_H^F} \kappa - \widehat{T}_L - \frac{\widehat{\pi}_L^F}{1 + \widehat{\pi}_L^F} \kappa \right) \quad (32)$$

to ensure that the government chooses to default, per effects of self-validating expectations of fiscal stress, also in the high output state.

3.4 Multiple equilibria and macroeconomic resilience

From the description of the economy and the optimal policy plans above, it is far from clear that a “printing press” alters the mechanism by which the economy is vulnerable to self-fulfilling run on debt. Indeed, the following two propositions, in analogy to propositions 1 and 2, states that the option to monetize debt — aiming at reducing the ex-post value of debt via inflation — and raise seigniorage revenues does not shield a country from confidence crises. Setting $\xi_\theta = 0$ (no fixed output costs of default) we so write the analog of proposition 1 for our monetary economy.

Proposition 3 *In the economy summarized by (18), (19), and (28), with the government optimally choosing taxes satisfying either (21) or (23) in case of default, or (25) otherwise, setting $\xi_\theta = 0$, under the maintained assumptions (29), (30), (31) and (32), the equilibrium will exist and will not be unique. There will be a fundamental equilibrium in which default will occur only the low output state of the world, with the equilibrium haircut given by $\theta_H^F = 0$ and*

$$0 < \widehat{\theta}_L^F = \frac{RB \frac{\mu(1 + \widetilde{\pi}_H) + (1 - \mu)(1 + \widehat{\pi}^F)}{1 + \widehat{\pi}^F} + G - \widehat{T}_L - \frac{\widehat{\pi}^F}{1 + \widehat{\pi}^F} \kappa}{\left[(1 - \alpha) RB \frac{\mu(1 + \widetilde{\pi}_H) + (1 - \mu)(1 + \widehat{\pi}^F)}{1 + \widehat{\pi}^F} - (1 - \mu) \left[\widehat{T}_L + \frac{\widehat{\pi}^F}{1 + \widehat{\pi}^F} \kappa - G \right] \right]} < 1, \quad (33)$$

the trade-off between taxation and inflation given by

$$\begin{aligned} z'(\widetilde{T}_H, Y_H) \left(B\widetilde{R}^F + \kappa \right) &= (1 + \widetilde{\pi}_H)^2 \mathcal{C}'(\widetilde{\pi}_H) \\ \frac{\alpha}{1 - \alpha} \left(B\widetilde{R}^F + \kappa \right) &= (1 + \widehat{\pi}^F)^2 \mathcal{C}'(\widehat{\pi}^F) \end{aligned}$$

and the ex-ante interest rate determined as follows

$$\tilde{R}^F = \frac{\mu(1 + \tilde{\pi}_H) + (1 - \mu)(1 + \hat{\pi}^F)}{\mu + (1 - \mu)(1 - \hat{\theta}_L^F)} R \quad (34)$$

There will be another equilibrium, driven by self-validating expectations, where the default rate, the tax rate and the inflation rate in each state are given by the solution to the following system

$$\begin{aligned} \hat{T}_H - G - \left(1 - \hat{\theta}_H^N(1 - \alpha)\right) \frac{\tilde{R}^N}{1 + \hat{\pi}_H^N} B + \frac{\hat{\pi}_H^N}{1 + \hat{\pi}_H^N} \kappa &= 0 \\ \hat{T}_L - G - \left(1 - \hat{\theta}_L^N(1 - \alpha)\right) \frac{\tilde{R}^N}{1 + \hat{\pi}_L^N} B + \frac{\hat{\pi}_L^N}{1 + \hat{\pi}_L^N} \kappa &= 0 \\ 0 &\leq \hat{\theta}_L^N \leq 1 \end{aligned}$$

$$\tilde{R}^N \left[\mu(1 - \hat{\theta}_H^N) + (1 - \mu)(1 - \hat{\theta}_L^N) \right] = \left[\mu(1 + \hat{\pi}_H^N) + (1 - \mu)(1 + \hat{\pi}_L^N) \right] R$$

and either

$$\frac{\alpha}{1 - \alpha} (B\tilde{R}^N + \kappa) = (1 + \hat{\pi}^N)^2 \mathcal{C}'(\hat{\pi}^N)$$

if the constraint $\theta_L^N \leq 1$ is not binding,

$$z'(\bar{T}_L, Y_L) (B\tilde{R} + \kappa) = (1 + \pi_L)^2 \mathcal{C}'(\pi_L)$$

otherwise.

Proof. See appendix ■

Including fixed output costs of default prevents multiplicity for a low stock of initial debt, as in proposition 2.

Proposition 4 *In the economy described by proposition 3, for given fixed output costs of default ($\xi_\theta > 0$):*

(a) *Equilibrium is unique if the government debt B is sufficiently low so that (27) holds in the high-output state, namely B satisfies the following inequality:*

$$\begin{aligned} \xi_\theta \geq & z(G + \frac{\tilde{R}^N B - \tilde{\pi}_H \kappa}{1 + \tilde{\pi}_H}, Y_H) - z(\hat{T}_H, Y_H) + \left[\mathcal{C}(\tilde{\pi}_H) - \mathcal{C}(\hat{\pi}^N) \right] - \\ & - \frac{\alpha}{1 - \alpha} \left[\frac{\tilde{R}^N B - \hat{\pi}^N \kappa}{1 + \hat{\pi}^N} - (\hat{T}_H - G) \right] + \left(\frac{1 + \hat{\pi}^N}{1 + \tilde{\pi}_H} - 1 \right) \frac{\tilde{R}^N}{1 + \hat{\pi}^N} B \end{aligned}$$

where

$$\begin{aligned} \tilde{R}^N &= \frac{(1 + \hat{\pi}^N)}{\mu(1 - \theta_H^N) + (1 - \mu)(1 - \theta_L^N)} R \\ \frac{\alpha}{1 - \alpha} (B\tilde{R}^N + \kappa) &= (1 + \hat{\pi}^N)^2 \mathcal{C}'(\hat{\pi}^N) \end{aligned}$$

In this unique equilibrium, default may or may not be chosen by the government in the low-output state, depending on whether the level of cost satisfies the following inequality

$$\xi_\theta < z\left(G + \frac{\tilde{R}^F B - \tilde{\pi}_L \kappa}{1 + \tilde{\pi}_L}, Y_L\right) - z(\hat{T}_L, Y_L) + \left[\mathcal{C}(\tilde{\pi}_L) - \mathcal{C}(\hat{\pi}^F)\right] - \frac{\alpha}{1 - \alpha} \left[\frac{\tilde{R}^F B - \hat{\pi}^F \kappa}{1 + \hat{\pi}^F} - (\hat{T}_L - G) \right] + \left(\frac{1 + \hat{\pi}^F}{1 + \tilde{\pi}_L} - 1 \right) \frac{\tilde{R}^F}{1 + \hat{\pi}^F} B$$

(b) there are two equilibria, characterized as in proposition 3, if the government debt B is sufficiently large so that (27) is violated in the high-output state.

Proof. See appendix. ■

Together, the two propositions above suggest that, relative to the real economy studied in the previous section, debt-monetization and seigniorage obviously affect the equilibrium policy trade-offs. But *per se* printing money does not rule out multiplicity.²⁶ The reason is straightforward: inflation is not costless from a macroeconomic perspective, and it will be set optimally in relation to the costs involved by raising taxes and/or defaulting.

Multiplicity is actually of exactly the same kind as in the real economy: partial repudiation via haircuts differs across equilibria, but, for a given default rate θ , the inflation rate is uniquely determined — there is no multiplicity in debt monetization. In this respect, our results differ from those in Calvo (1988), who also provides an example of monetary economy with multiple equilibria and self-fulfilling expectations of default. The difference depends on two crucial features of the model economy. First, in the monetary version of our model the government may still choose to impose haircuts on the holders of public debt — a possibility that is instead ruled out by assumption in the monetary economy studied by Calvo. Second, inflation has standard convex costs. In contrast, Calvo (1988) specifies non-standard costs $C(\pi)$, implying multiplicity in the rate of inflation itself.²⁷ Uniqueness of the inflation rate is an important result for the analysis in the rest of our paper, where we study conditions under which central banks can provide a backstop to government debt and rule out self-fulfilling runs.

An important question raised by a comparison of propositions 2 and 4 is whether, even if ineffective to rule out self-fulfilling crises, monetary sovereignty may nonetheless increase macro resilience to them. This would be the case if the stock of debt for which the equilibrium is unique were necessarily higher in a

²⁶Note that, as for the real economy in the previous section, by virtue of the regularity conditions we impose on the size of debt relative to the tax capacity of the country, default is always partial in the low-output state in the F-equilibrium, as well as in the high-output state in the N-equilibrium.

²⁷Our model would also predict multiplicity in inflation rates, if we replaced our assumptions about $C(\pi)$ with the one in Calvo (1988).

monetary economy (everything else equal) than in an economy without inflation-related benefits (seigniorage) and distortions. Addressing this question clarifies that ‘monetization’ has two opposing effects on the decision to default. Consider (27) evaluated at $\kappa = C(\pi) = \pi = 0$, determining the threshold stock of debt that marks the switch from equilibrium uniqueness to multiplicity in the real economy. At that point, some revenue from the inflation tax allows the government to reduce taxation and the associated loss of output. Through this channel, inflation raises the level of nominal debt at which the switch occurs. However, there are now output costs due to inflation. A large differential between inflation costs without and with default tends to lower the switching threshold. If seigniorage revenue turns out to be low in equilibrium, it may be possible that equilibrium multiplicity becomes a problem for a lower stock of initial debt.²⁸

4 Policy options to stem self-fulfilling runs on debt

When multiple equilibria are possible, differences in welfare across equilibria are driven by output losses caused by taxation and default. Specifically, the increase in the interest rate due to self-fulfilling expectations causes unnecessary output disruption due to the combined effect of higher taxation, inflation and default not only in the low-output state, but also in the high output (normal) state.

The fact that equilibria with non-fundamental default are detrimental to social welfare raises the issue of what kind of policies can be deployed to rule it out. As emphasized by Calvo (1988), there is a straightforward policy that can improve welfare. As is well understood, self-fulfilling debt crises could be prevented by an institution that, in period 1, would credibly set a ceiling \bar{R} on the interest rate, at which it stands ready to buy any amount of government debt. The ceiling \bar{R} should be sufficiently low as to rule out the bad equilibrium driven in part by self-fulfilling expectations, and high enough to avoid ex-ante

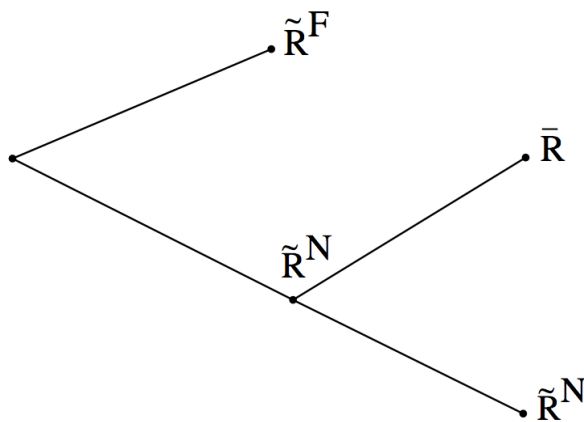
²⁸Observe that the budget costs of debt default are independent of inflation, because the state-contingent monetization of the debt (relevant for their calculation) is indeed perfectly anticipated by agents.

losses.²⁹ In our economy of section 2, this would imply:

$$\tilde{R}^N > \bar{R} \geq \tilde{R}^F = \frac{R}{1 - (1 - \mu)\theta_L^F}.$$

Such ceiling would essentially coordinate market expectations on the fundamental equilibrium only. This is because knowing that interest rates cannot rise to the level \tilde{R}^N , the only market equilibrium is one in which private agents find it optimal to bid for the government debt at the lower equilibrium rate \tilde{R}^F . As a result, there is no need for the institution to actually purchase any amount of debt. The argument is summarized in the simple game depicted in Figure 3 below.

p



If the lender of last resort is expected to play \bar{R} at the node \tilde{R}^N , the latter rate cannot be a market equilibrium.³⁰

²⁹It is easy to verify that the ceiling cannot exceed the market rate at which the best response of the government is a strictly positive default rate in the high output state. Note that, while the ceiling on interest rates should be sufficiently low as to rule out the bad equilibrium driven in part by self-fulfilling crises, it should also be at least as high as the interest rate in the fundamental equilibrium in order to avoid a transfer of resources covering the short-fall of fiscal revenues under weak fundamentals – too low an interest rate would effectively amount to a bailout. This would occur if the cap rate were to be set *below* the rate in the fundamental equilibrium:

$$\tilde{R}^N > \bar{R} \geq \tilde{R}^F > R^{bail-out}$$

Of course, anticipations of a bailout of this kind is a distortion, creating all sort of destabilizing incentives ex-ante, giving rise to “moral hazard” (see e.g. Green 2010 and Prescott 2010 for a recent discussion).

³⁰In Corsetti and Dedola (2011), we show that, in contrast to a transfer implicit in an

4.1 Interventions in the sovereign debt market

A key feature of the intervention policy is that, to be effective, the rate cap should be fully credible. In other words, private agents must believe that, if they coordinated on the bad equilibrium, the intervening institution would have no incentive to deviate from the announced policy of buying debt at \bar{R} , effectively playing the role of lender of last resort.

Doubts about the implementation of the rate cap would obviously undermine the success of the policy. In principle, one could just assume that the lender of last resort can commit to the pre-announced policy. It is however more realistic and interesting to explore the determinants of its behavior. To make interventions a sustainable belief, two conditions need to be satisfied: interventions must be (i) feasible (the lender of last resort must have sufficient resources) and (ii) welfare improving from the perspective of the intervening institution. Assuming that such an institution is benevolent, this means that domestic welfare must be higher at \bar{R} than at \tilde{R}^N . So, a benevolent lender of last resort implies that \bar{R} must be sufficiently lower than \tilde{R}^N , as not to induce the government to default in the high state.

Another key feature stressed in the debate on the lender of last resort concerns the size and nature of interventions. The view just discussed emphasizes the idea that a credible threat to purchase debt up to the entire stock of public debt would be sufficient to coordinate markets on the fundamental equilibrium (see De Grauwe 2011 for a recent application of this argument to the European Central Bank). A different view emphasizes the need to carry out interventions in the debt market, related to a subtly different target. According to this view, the lender of last resort should actually engage in debt purchases in response to an incipient self-fulfilling run, with the goal of steering the economy towards a better equilibrium — characterized by a market interest rate closer to the rate in the fundamental equilibrium. In terms of the above Figure 3, rather than the rate at which the lender of last resort ‘shocks and awes’ the markets by threatening to buy all debt (off-equilibrium), \bar{R} would denote a new equilibrium interest rate (at which agents would still buy some debt), resulting from the intervention of the lender of last resort.

Which institution qualifies for carrying out a successful intervention policy? Clearly, a government unable to commit to future policies (as we have assumed in our analysis so far) cannot pursue this strategy on its own. If investors believe there will be default, they will simply refuse to buy debt at a price inconsistent with their expectations, independently of any government announcement. A natural candidate would rather be a deep-pocket external public institution, such as the International Monetary Fund — a full analysis requiring the specification of this institution’s objectives and budget constraint (see e.g. Corsetti, Guimaraes, Roubini 2005 and Morris and Shin 2006, and Zwart 2007 among others).

intervention rate below the fundamental rate, liquidity support does not discourage costly reforms that improve government budget (see also Corsetti et al 2005 and Morris and Shin 2006).

4.2 The role of central banks

The question we want to address in the rest of the paper is whether the central bank can rule out self-fulfilling runs on sovereign debt under any circumstances. What makes this question particularly intriguing is that, from an aggregate perspective, any purchase of government debt by the monetary authorities is at best backed by their consolidated budget with the fiscal authorities — i.e. there are no additional resources to complement tax and seigniorage revenues.

However, central banks do appear to differ from governments in at least two respects. The first one concerns the ability to commit to state contingent policies — raising the question of whether the central bank would be able to eliminate self-fulfilling debt runs by committing to an optimal (state-contingent) inflation plan. A second difference concerns a central bank’s ability to swap government debt for monetary liabilities that are not exposed to outright default.³¹ We consider the first point below — the second point will be the subject of the next section.

Using the model specification of Section 3, we now assume a policy scenario in which the fiscal authority still acts under discretion, but the benevolent central bank can credibly commit to a state-contingent inflation policy. In our setting, this implies that inflation will be chosen before agents form expectations and bid for the government debt at the interest rate \bar{R} (see Persson and Tabellini 1993). For simplicity of exposition, we posit $\mu \rightarrow 1$, so that there is no fundamental fiscal stress in the economy. Under policy discretion, multiplicity of course still obtain with $\mu \rightarrow 1$, as a special case of propositions 3 and 4.

Under the hypothesis that the central bank can commit, the marginal conditions governing the choice of inflation become, respectively:

$$(1 + \pi_H^F)^2 \mathcal{C}'(\pi_H^F) = z'(T_H, Y_H) \kappa \quad (35)$$

in the fundamental equilibrium without default, and

$$(1 + \pi_H^N)^2 \mathcal{C}'(\pi_H^N) = -\kappa, \quad (36)$$

conditional on (non-fundamental) default, if any. Note that, contrary to the analysis in Section 3, in the fundamental equilibrium inflation is positive only to the extent that seigniorage revenue is optimally traded-off against distortionary taxation — there is no systematic attempt by the central bank to resort to surprise inflation (compare the above expressions with equation (26)). Indeed, conditional on non-fundamental default, the optimal inflation rate could even be negative to support consumption by increasing the real value of money. Moreover, as apparent from the above expressions, the optimal inflation is decreasing in the demand for real balances κ — capping the seigniorage revenue (seigniorage and thus inflation is zero for $\kappa \rightarrow 0$).

³¹See Gertler and Karadi (2011) for an analysis of ‘unconventional monetary policy’ by which central banks exploit their advantage in issuing riskless liabilities to act as financial intermediaries during financial crises, providing funds to private firms.

By virtue of commitment, however, under certain conditions the central bank may be able to rule out multiplicity. Namely, the central bank may find it optimal to threaten to raise inflation and seigniorage if a speculative run on debt drives the interest rate away from the fundamental value — with the result of undermining \tilde{R}^N as an equilibrium outcome. It can be shown that such a threat can indeed be part of the optimal inflation policy of the central bank under two strict conditions. First, debt cannot be too high relative to seigniorage revenue, so that the budget constraint would still be satisfied under inflationary financing (note that if $\kappa = 0$, seigniorage and thus optimal inflation would always be zero independently of default); second, the fixed output costs of default are large enough relative to the costs of inflation, so that inflationary financing would be welfare-improving over the N-equilibrium. Holding these conditions, indeed, the benefits from increased inflation and seigniorage (mitigating the need for raising taxes) in terms of avoiding the output losses due to default, would largely exceed the costs of inflation. Yet, the range of applicability of this result is rather narrow.

5 A model of debt default and central bank interventions in the debt market

A second, distinct feature of central banks consists of their ability to issue monetary liabilities that are nominally default-free — in modern economies, high powered money include cash and especially bank reserves, often interest-bearing, which are clearly exposed to inflation risk, but not to outright default risk. This ability may follow from commitment, or may reflect very high costs of default on assets at the core of the financial system. In our analysis below, we do not explore these possible explanations, but simply posit the assumption that, while government debt is exposed to the risk of default via both outright haircuts and inflation, central bank liabilities such as high powered money are subject only to ex-post inflation risk. Based on this assumption, we work out conditions under which the central bank can carry out successful interventions policy in the debt market, without compromising its own budget constraint and welfare objectives.

We argue that, while constrained, it is this specific features of monetary liabilities that may enable a central bank to redress the problem of equilibrium multiplicity in the debt market. Without assuming commitment to a state-contingent inflation policy (as we did at the end of the previous section), we will let the central bank buy government debt in the initial period, by issuing “reserves” that are remunerated at a market rate, rather than fiat money. These interventions have no consequences on the period-one price level, consistent with the idea that a central bank backstop to the government does not need to have immediate inflationary consequences³² — although it may raise expectations of

³²It is also consistent with our assumption that money and the price level are predetermined in the initial period.

future inflation, to the extent that the central bank is expected to honour its nominal liabilities via seigniorage revenue and/or the ‘printing press’.

For the central bank nominal liabilities to be default-free, indeed, it is crucial that they are effectively backed under all contingencies. Generally, backing would require more than the mere resort to the printing press. This is so because, on the one hand, the revenue from seigniorage is typically quite moderate, and thus inadequate to guarantee interventions beyond a very limited scale, while a systematic resort to the printing press to inflate away debt cannot be but anticipated by the markets, hence arguably reflected in higher interest rates on all public liabilities. On the other hand, even if seigniorage and ex-post inflation surprise could provide enough resources, it is far from clear that letting inflation adjust residually (de facto relying only on one instrument, rather than optimizing across all available instruments) would be desirable from a welfare perspective.

The question once again pertains to the interactions between fiscal and monetary authorities. Consistent with the approach taken so far in the paper, we first focus on the benchmark case in which these authorities pursue the same objective — maximizing the residents’ welfare ex post — subject to their consolidated budget constraints. We then assess the consequences of imposing the constraint that transfers from the central bank to the government be non-negative — de facto making the central bank fully responsible for servicing its own liabilities — which may arise in case of institutional or political conflict among policymakers.

Through the lens of our default model, we can identify the precise conditions which can make the central bank an effective lender of last resort, either via a credible threat to satisfy the entire financial need of the government, so to rule out self-fulfilling runs — a “shock and awe” strategy —; or via purchases of government paper — a “fine-tuned backstop” strategy —, bringing the economy onto a better equilibrium than the self-fulfilling one. The problem at hand is particularly compelling when there is a positive probability of a fundamental default, causing a fiscal shortfall and thus potential losses on the debt owned by the central bank ex-post. For the central bank to be able to honor its own liabilities, either taxes or seigniorage, or possibly both, must adjust, in line with the classical analysis by Sargent and Wallace (1981). In this framework, claiming that central bank’s liabilities are default free in nominal terms because they can always be repaid by printing money, is tantamount to assume that only inflation must adjust residually, to cover any loss borne by the central bank.

5.1 Budget constraints

In what follows, we reconsider our model in Section 3, allowing the central bank to purchase a fraction ω of the outstanding stock of debt issuing nominal assets \mathcal{H} — “reserves” — free from the risk of outright default, at some pre-announced rate \bar{R} , if market interest rates rise to the non-fundamental level \tilde{R}^N . The assets \mathcal{H} issued by the central bank is remunerated at the default-free

nominal rate $(1+i)$.³³ This implies that there are now two types of public liabilities that are relevant to our study of sovereign default: government debt and interest-bearing reserves. As discussed above, a key difference is that, while ex-post inflation surprises affect the real value of the outstanding stock of both (at the price of distortions induced by inflation), outright haircuts θ are applied to B only (at the price of output and budget costs, as discussed in the previous sections).

The balance sheet of the central bank in the first period and its budget constraint in the second period read:

$$\begin{aligned}\omega B &= \mathcal{H} \\ \mathcal{T}_i &= \frac{\pi_i}{1+\pi_i}\kappa + \frac{(1-\theta_i)}{1+\pi_i}\omega B\bar{R} - \frac{(1+i)}{1+\pi_i}\mathcal{H} = \\ &= \frac{\pi_i}{1+\pi_i}\kappa + \left(\frac{(1-\theta_i)}{1+\pi_i}\bar{R} - \frac{(1+i)}{1+\pi_i}\right)\omega B\end{aligned}\tag{37}$$

where \mathcal{T}_i are transfers from the central bank to the fiscal authority (recall that we have imposed $M_1 \rightarrow 0$ for analytical convenience).

In order to write the budget constraint of the fiscal authority, two issues need to be addressed. First, the government may attempt to default selectively on the central bank as opposed to private investors. Second, the cost of default may in principle be different if debt is held by the central bank, as opposed to investors only. In what follows, for clarity of exposition we proceed under two simplifying assumptions. First, we posit that the government cannot discriminate the central bank's from agents' holding of debt when applying the haircut θ_i . Second, defaulting on the central bank is no more costly than defaulting on investors. Abstracting from fixed costs, i.e. setting $\xi_\theta = 0$ (as in proposition 1 and 3), we posit $0 \leq \alpha_{CB} \leq \alpha$. Under these assumptions, the government budget constraint in each state of nature (H or L) in period 2 then reads:

$$T_i - G = [1 - \theta_i(1 - \alpha)] \frac{\bar{R}}{1 + \pi_i} (1 - \omega) B + [1 - \theta_i(1 - \alpha_{CB})] \frac{\bar{R}}{1 + \pi_i} \omega B - \mathcal{T}_i\tag{38}$$

where $\alpha, \alpha_{CB}, \theta_i \in [0, 1]$. Consolidating the budget of the fiscal and the monetary authorities yields the following key expression:

$$T_i + \frac{\pi_i}{1 + \pi_i} \kappa - G = [1 - \theta_i(1 - \alpha)] \frac{\bar{R}}{1 + \pi_i} (1 - \omega) B + \left[\alpha_{CB} \frac{\bar{R}}{1 + \pi_i} + \frac{(1 + i)}{1 + \pi_i} \right] \omega B\tag{39}$$

Ultimately, the primary surplus cum seigniorage finances both the interest payments by the government to private investors (net of default but gross of the transaction costs associated to it); *and* the interest bill of the central bank — always paid in full under our assumptions. The consolidated budget constraint

³³One way to think about this approach is as sterilized interventions that do not change the amount of “liquidity” (M in our model) in the economy, with no consequences for current inflation.

so clarifies that the purchase of government debt financed by issuing reserves today does not mechanically translate into higher inflation in the future. It raises inflation only to the extent that, after repaying the bonds in the hands of private investors (net of default but gross of transaction costs), the primary surplus falls short of the interest bill on reserves at the desired level of inflation (and thus at the desired seigniorage level).

The budget constraint of the representative agent is

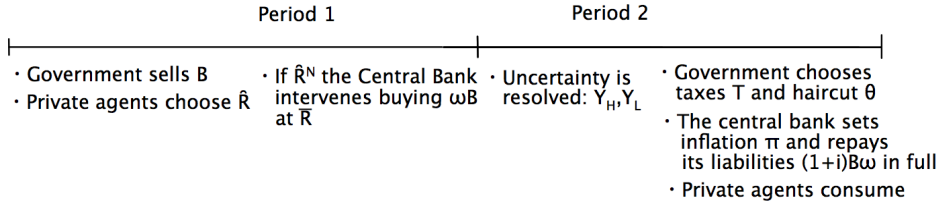
$$C_i = [Y_i - z(T_i, Y_i)] + KR - T_i + (1 - \theta_i)(1 - \omega) \frac{B}{1 + \pi_i} \bar{R} + \frac{(1 + i)}{1 + \pi_i} \mathcal{H} - \frac{\pi_i}{1 + \pi_i} \kappa - \mathcal{C}(\pi_i),$$

Combining the three constraints above we can write the objective function of benevolent policymakers:

$$\begin{aligned} C_i &= [Y_i - z(T_i, Y_i)] + KR + & (40) \\ & - T_i + (1 - \theta_i)(1 - \omega) \frac{B}{1 + \pi_i} \bar{R} - \mathcal{T}_i + \frac{\pi_i}{1 + \pi_i} \kappa + \frac{(1 - \theta_i)}{1 + \pi_i} \omega B \bar{R} - \frac{\pi_i}{1 + \pi_i} \kappa - \mathcal{C}(\pi_i) \\ & = [Y_i - z(T_i, Y_i)] - G - \theta_i [(1 - \omega) \alpha + \omega \alpha_{CB}] \frac{\bar{R}}{1 + \pi_i} B - \mathcal{C}(\pi_i). \end{aligned}$$

The timeline is summarized by the figure below.

Timeline: a model with central bank interventions in the debt market



5.2 Optimal policy under discretion

The discretionary policy plan by benevolent policymakers taking the central bank intervention policy (ω and \bar{R}) and the interest rate i set in period 1 as given is as follows. *Conditional on default*, the upper bound on the country's willingness to raise distortionary taxes is:

$$z'(\hat{T}_i, Y_i) = \frac{\alpha - (\alpha - \alpha_{CB})\omega}{1 - [\alpha - (\alpha - \alpha_{CB})\omega]}. \quad (41)$$

The maximum primary surplus, net of the inflation tax, that the country finds it optimal to generate in the second period, $\widehat{T}_i - G$, (and the associated net output $Y_i - z(\widehat{T}_i)$) are now a function of ω and the difference between α and α_{CB} . Insofar as the variable costs of default falls with debt purchases by the central bank, so does the optimal taxation \widehat{T} . While central bank purchases do affect the incentives to increase taxes relative to imposing a higher haircut θ_i , for $\theta_i = 1$, taxes will still have to adjust to satisfy the budget constraint (at the equilibrium level of transfers from the central bank \mathcal{T}_i , and inflation π_i)

$$\begin{aligned} \mathcal{T}_i - G &= [1 - \theta_i(1 - \alpha + (\alpha - \alpha_{CB})\omega)] \frac{\overline{R}}{1 + \pi_i} B - \mathcal{T}_i & (42) \\ \widehat{T}_i &\leq \mathcal{T}_i = G + [\alpha - (\alpha - \alpha_{CB})\omega] \frac{\overline{R}}{1 + \pi_i} B - \mathcal{T}_i, \quad i = L, H \end{aligned}$$

Conditional on no default, given seigniorage revenues, taxes must be raised to cover total public spending.

The optimal inflation rate (associated with the optimal tax rates) is given implicitly by the following two equations, one conditional on default:

$$\frac{\alpha(1 - \omega) + \omega\alpha_{CB}}{1 - [\alpha(1 - \omega) + \omega\alpha_{CB}]} (B\overline{R} + \kappa + [(1 + i) - (1 - \theta_i)\overline{R}] \omega B) = (1 + \widehat{\pi})^2 \mathcal{C}'(\widehat{\pi}), \quad (43)$$

the other conditional on no default ($\theta_i = 0$):

$$z'(\widetilde{T}_i, Y_i) (B\overline{R} + \kappa + [(1 + i) - \overline{R}] \omega B) = (1 + \widetilde{\pi}_i)^2 \mathcal{C}'(\widetilde{\pi}_i). \quad (44)$$

It is easy to verify that, for $\omega = 0$, these optimality conditions are the same as in Section 3.

5.3 Equilibrium debt pricing

With two public liabilities, there are two equilibrium interest parity conditions. The interest rate on reserves $1 + i$, free from the outright default risk, is priced by private agents based on expected inflation:

$$(1 + i) = [\mu(1 + \pi_H) + (1 - \mu)(1 + \pi_L)] R. \quad (45)$$

Moreover, to the extent that the central bank does not buy the whole stock of outstanding debt, by no-arbitrage it must be the case that the price of government debt to the price of the central bank's liabilities are also linked to each other by the following interest parity condition:

$$(1 + i) = \overline{R} [\mu(1 - \theta_H) + (1 - \mu)(1 - \theta_L)] \quad \text{if } 0 < \omega < 1. \quad (46)$$

These expressions emphasize that interest rates in period 1 are rising in expectations of both inflation and default.

5.4 A comparative analysis of intervention strategies

In characterizing the optimal policy above, the interest rates \bar{R} and i and the central bank purchases ω are treated as predetermined. The question we address in this section concerns the characterization of intervention policies $\{\omega$ and $\bar{R}\}$ that are feasible and welfare-improving over the N-equilibrium. As argued above, these conditions are necessary for central bank interventions to be effective in ruling out multiplicity.

As already mentioned, we will contrast alternative intervention strategies — “shock and awe” versus “finetuned backstop” —, studying the benchmark case in which both policymakers act cooperatively subject to the consolidated budget constraint, as well as a key deviation from this benchmark, assuming that the central bank operates under the constraint that its transfer to the government are non-negative.

5.4.1 ‘Shock and awe’: the central bank stands ready to underwrite government debt issuance in full

Not surprisingly, it is easy to show that, under fiscal and monetary cooperation, a “shock-and-awe” strategy — by which the central bank stands ready to underwrite government debt issuance entirely — is feasible and welfare-improving relative to the non-fundamental equilibrium allocation. Namely, there exists a \bar{R} such that all the budget constraints and the first-order conditions for the optimal discretionary policy plan spelled out in the previous subsection are satisfied, and welfare is higher than in the N-equilibrium.

The intuition is straightforward: the central bank is able to swap risky government liabilities with riskless liabilities on which no discrete default is expected ex post — in practice redressing the government lack of commitment to service its debt, and allowing it to borrow at a rate even lower than \bar{R}^F .

However, this result is a bit too powerful, and begs the question: how come the central bank does not actually buy (instead of threatening to buy) the entire government debt under all circumstances? A key issue is that, once the entire stock of public debt is held by the monetary authority, the government arguably faces much lower economic costs of default, hence it may have stronger incentives to act opportunistically, up to placing the burden of adjustment mostly or even entirely on the central bank.

Indeed, consider the limit but plausible case with $\alpha_{CB} = 0$. The consolidated budget constraint simplifies to

$$\begin{aligned} T_i - G &= \frac{(1+i)}{1+\pi_i} B - \frac{\pi_i}{1+\pi_i} \kappa = \\ &= \begin{cases} \frac{[\mu(1+\pi_H) + (1-\mu)(1+\pi_L)]}{1+\pi_H} RB - \frac{\pi_H}{1+\pi_H} \kappa \\ \frac{[\mu(1+\pi_H) + (1-\mu)(1+\pi_L)]}{1+\pi_L} RB - \frac{\pi_L}{1+\pi_L} \kappa \end{cases}, \end{aligned}$$

since it would be optimal to set $\theta_i = 1$, in both the high- and the low-output state. Now, if the central bank were to be held responsible for backing its own

liabilities using the printing press in all states of the world (i.e. no adjustment were to be expected in the primary surplus $T_i - G$), the threat to fully backstop all outstanding debt would hardly be credible. First, seigniorage revenue is bounded, i.e., in our model $\lim_{\pi \rightarrow \infty} \frac{\pi_i}{1+\pi_i} \kappa = \kappa$, and inflation would be anticipated by rational agents: unless κ were implausibly large, or B implausibly small to create a situation of fiscal stress, seigniorage and debt monetization would arguably be insufficient for the central bank to repay $(1+i)H$ in full. Second, to the extent that fiscal and monetary authorities are benevolent, they would be willing to use all instruments, taxes, default and inflation, with the objective to minimize their combined distortions — with convex costs of inflation, using only the printing press may not improve over the N-equilibrium allocation.

Now, opportunistic governments facing little or no punishment for choosing a complete default over central bank held debt (i.e. $\alpha_{CB} = 0$) would set taxes and spending to match their own private welfare objectives, flaunting fiscal prudence. If this were the case, our model shows that, it would be quite difficult to find a *feasible* \bar{R} ensuring a welfare improvement over the run debt-run N-equilibrium.

Under this assumption, a “shock-and-awe” strategy could nonetheless be credible for some value of α_{CB} sufficiently larger than zero, de facto moderating the incentives for the government to take advantage of the central bank exposure to public debt — a result that resounds with the current debate on the conditionality to be attached to the central bank engagement in the sovereign debt markets. A different solution to the problem is provided by the alternative intervention strategy, discussed below.

5.4.2 A fine-tuned backstop strategy

With benevolent policymakers and a consolidated budget constraint, it is also easy to find a feasible backstop strategy, consisting of a pair $0 < \omega < 1$ and an intervention rate \bar{R} , that would improve on the non-fundamental equilibrium. Relative to the ‘shock-and-awe’ case, the central bank would not simply threaten to intervene contingent on private sector coordination on a bad equilibrium. It will effectively purchase debt. Hence, the equilibrium will be subject to an additional no-arbitrage condition linking the backstop interest rate \bar{R} to the rate offered by the central bank on its liabilities, $1+i$. We should note here that it will be feasible to set \bar{R} arbitrarily close to the fundamental rate.

Such a strategy profoundly alters the transmission of interventions relative to the “shock-and-awe” one. The fact that a substantial fraction of debt always remain in the hand of the private sector imposes discipline on the fiscal authorities: despite the central bank interventions in the market, a default would still have substantial output and budget costs — hence reducing the incentives for a government to act opportunistically.

Indeed, even allowing for a separation of budget constraints — de facto insisting that the central bank should be the sole responsible for its liabilities — it is possible to prove that under a partial backstop strategy would work under general conditions. Formally:

Proposition 5 *An equilibrium under a backstop policy at which the central bank buys a positive fraction of debt $0 < \omega < 1$ without losses with $\bar{R} = \tilde{R}^F$, as characterized by the following equations:*

$$\begin{aligned}\tilde{R}^F &= \frac{\mu(1 + \tilde{\pi}_H) + (1 - \mu)(1 + \hat{\pi}^F)}{\mu + (1 - \mu)(1 - \hat{\theta}_L^F)} R = \frac{\mu(1 + \bar{\pi}_H) + (1 - \mu)(1 + \bar{\pi}_L)}{\mu + (1 - \mu)(1 - \bar{\theta}_L)} R \quad (47) \\ \bar{\theta}_L &= \frac{\tilde{R}^F - \mu(1 + \bar{\pi}_H) - (1 - \mu)(1 + \bar{\pi}_L)}{(1 - \mu)\tilde{R}^F}, \quad (48)\end{aligned}$$

$$\bar{T}_H = \frac{\bar{\pi}_H}{1 + \bar{\pi}_H} \kappa + \frac{(1 - \mu)\bar{\theta}_L}{1 + \bar{\pi}_H} \omega \tilde{R}^F B \geq 0 \quad (49)$$

$$\bar{T}_H - G = \frac{\tilde{R}^F}{1 + \bar{\pi}_H} (1 - \omega) B + \frac{(1 + i)}{1 + \bar{\pi}_H} \omega B - \frac{\bar{\pi}_H}{1 + \bar{\pi}_H} \kappa. \quad (50)$$

$$z'(\bar{T}_H, Y_H) \left(\kappa + (1 - (1 - \mu)\omega\bar{\theta}_L) B \tilde{R}^F \right) = (1 + \bar{\pi}_H)^2 \mathcal{C}'(\bar{\pi}_H)$$

$$\bar{T}_L = \frac{\bar{\pi}_L}{1 + \bar{\pi}_L} \kappa - \frac{\tilde{R}^F}{1 + \bar{\pi}_L} \mu \bar{\theta}_L \omega B \geq 0 \quad (51)$$

$$\hat{T}_L(\alpha, \omega) - G = \left[1 + ((\mu - \alpha)\omega - (1 - \alpha))\bar{\theta}_L \right] \frac{\tilde{R}^F}{1 + \bar{\pi}_L} B - \frac{\bar{\pi}_L}{1 + \bar{\pi}_L} \kappa, \quad (52)$$

$$\frac{\alpha(1 - \omega)}{1 - \alpha(1 - \omega)} \left(\kappa + [1 + \mu\bar{\theta}_L\omega] B \tilde{R}^F \right) = (1 + \bar{\pi}_L)^2 \mathcal{C}'(\bar{\pi}_L),$$

will exist if the following conditions hold:

$$z' \left(\frac{(1 - \omega)\tilde{R}^F + \mu(1 + \bar{\pi}_H) + (1 - \mu)(1 + \bar{\pi}_L)\omega}{1 + \bar{\pi}_H} B - \frac{\bar{\pi}_H}{1 + \bar{\pi}_H} \kappa + G, Y_H \right) \quad (53)$$

$$\left(\kappa + \left((1 - \omega)\tilde{R}^F + \mu(1 + \bar{\pi}_H) + (1 - \mu)(1 + \bar{\pi}_L)\omega \right) B \right) = (1 + \bar{\pi}_H)^2 \mathcal{C}'(\bar{\pi}_H) \quad (54)$$

$$\frac{\alpha(1 - \omega)}{1 - \alpha(1 - \omega)} \left(\kappa + \left[\tilde{R}^F + \mu \frac{\tilde{R}^F - \mu(1 + \bar{\pi}_H) - (1 - \mu)(1 + \bar{\pi}_L)}{(1 - \mu)} \omega \right] B \right) = (1 + \bar{\pi}_L)^2 \mathcal{C}'(\bar{\pi}_L) \quad (55)$$

$$\begin{aligned} \max \left\{ \frac{\tilde{R}^F B - (1 + \bar{\pi}_H) [\bar{T}_H - G] - \bar{\pi}_H \kappa}{\left[\tilde{R}^F - \mu(1 + \bar{\pi}_H) + (1 - \mu)(1 + \bar{\pi}_L) \right] B}, 0 \right\} < \omega \leq \\ (1 - \mu) \frac{\bar{\pi}_L \kappa}{\mu \left[\tilde{R}^F - \mu(1 + \bar{\pi}_H) - (1 - \mu)(1 + \bar{\pi}_L) \right] B} \quad (56) \end{aligned}$$

The conditions for which the proposition holds revolve around how sensitive are the (derivative of the) cost of distortionary taxes $z(T_i, Y_i)$, and the cost of inflation $C(\pi)$. Intuitively: first, the fraction of debt ω bought by the central bank should be low enough as to guarantee that the fiscal authority finds it desirable to increase taxes rather than defaulting. Second, in the low output state, the central bank will face losses on its sovereign debt holdings: the costs of inflation as well as ω must be low enough that these losses are covered by seigniorage in that state

$$\tilde{R}^F \mu \bar{\theta}_L \omega B \leq \bar{\pi}_L \kappa.$$

It is worth emphasizing that, even if the central bank targets a sovereign interest rate that is the same as in the F-equilibrium, \tilde{R}^F , the allocation is not the same as in that equilibrium. Namely, with central bank interventions, in the high output state both \bar{T}_H and $\bar{\pi}_H$ will be lower than their counterparts in the fundamental equilibrium \tilde{T}_H and $\tilde{\pi}_H$ — reflecting the fact that the purchase of debt by the central bank (for $0 < \omega < 1$) lowers the interest rate bill of the government, for $(1 + i) < \tilde{R}^F$.³⁴

6 Conclusions

This paper has analyzed the interactions between government default and monetary sovereignty in a closed economy framework. However, its results are relevant also for the current debate on sovereign default in a multi-country monetary union.

The analysis makes it clear that it is neither monetary sovereignty *per se*, nor a central bank commitment to inflation that matters in the face of self-fulfilling debt crises. Rather it is the willingness of the central bank to backstop the fiscal authority, with the understanding that losses related to this activity will be covered by the latter. Absent this understanding, what matters is the size of seigniorage relative to the size of the backstop.

The lessons for a currency area are apparent. Countries in a monetary union could be more vulnerable to debt crises when the common central bank cannot count on the joint support of all national fiscal authorities. In this case the central bank will have to weigh the benefit of providing the backstop to a subset of countries, at the cost of using the seigniorage accruing to all countries in the union, potentially associated with a higher area-wide inflation rate.

³⁴Under our assumption, θ is the same across agents and the central bank. A different possibility is that a different θ could be applied to central bank holdings of public debt. Under the maintained assumption that α_{CB} is zero, or simply lower than α , the conditions for a credible backstop analyzed in this subsection would obviously become more stringent.

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7 Appendix

to be added.