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False News, Informational Efficiency, and Price Reversals*

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Abstract

Speculators can discover whether a signal is true or false by processing it but this takes time. Hence they face a trade-off between trading fast on a signal (i.e., before processing it), at the risk of trading on false news, or trading after processing the signal, at the risk that prices already reflect their information. The number of speculators who choose to trade fast increases with news reliability and decreases with the cost of fast trading technologies. We derive testable implications for the effects of these variables on (i) the value of information, (ii) patterns in returns and trades, (iii) the frequency of price reversals in a stock, and (iv) informational efficiency. Cheaper fast trading technologies simultaneously raise informational efficiency and the frequency of “mini-flash crashes”: large price movements that revert quickly.

Keywords: news, high-frequency trading, price reversals, informational efficiency, mini-flash crashes.

JEL classification: G10, G12, G14

Résumé

Les spéculateurs peuvent découvrir si un signal est vrai ou faux en le traitant mais ceci prend du temps. Par conséquent, ils font face à un choix entre l'utilisation rapide d'un signal pour une transaction (c'est-à-dire avant de le traiter), au risque de se baser sur une fausse nouvelle, ou bien réaliser la transaction après le traitement du signal, au risque que les prix reflètent déjà l'information. Le nombre de spéculateurs qui choisissent de réaliser des transactions rapidement augmente avec la fiabilité des nouvelles et diminue avec le coût des technologies de transaction rapide. Nous tirons des implications testables pour les effets de ces variables sur (i) la valeur de l'information, (ii) les dynamiques des rendements et des transactions, (iii) la fréquence des retournements de prix pour un titre, et (iv) l'efficacité informationnelle. Des technologies de négociation rapide à meilleur marché relèvent simultanément l'efficacité informationnelle et la fréquence des “mini cracks boursiers”: de grands mouvements de prix qui se retournent rapidement.

Mots-clés: nouvelles, trading haute fréquence, retournements de prix, efficacité informationnelle, mini cracks boursiers.

Classification JEL: G10, G12, G14

Non Technical Summary

Improvement in trading technologies enable speculators to react to news in a few milliseconds. However, in reacting fast to information, speculators take the risk of trading on false news as information processing (e.g., checking news accuracy) takes time. And, indeed, market participants claim that large sudden price drops or spikes followed by quick price reversals are increasingly common and have nicknamed these patterns “mini flash crashes”.

Mini-flash crashes are a source of concerns as they seem symptomatic of market fragility and informational inefficiency. In particular, sharp price drops in one asset might propagate to other assets leading to market-wide disruptions, as observed during the 2010 flash crash. Media, practitioners, and regulators link mini-flash crashes and more generally market instability to the growth of computerized and fast trading (the so called “race to zero”)

However, academic studies do not support the view that computerized fast trading is associated with less efficient markets. For instance, using Nasdaq data, Brogaard, Hendershott, and Riordan (2013) conclude that high-frequency traders contribute to price discovery. Boehmer et al.(2013) reach a similar conclusion in a cross-country study.

This disconnect between anecdotal and academic evidence raise the following question: can faster trading on information make financial markets simultaneously more informationally efficient *and* unstable? Our goal in this paper is to study this question, building up on the idea that speculators might receive false signals and that processing signals (e.g., checking the validity of a signal by obtaining additional signals) takes time. Importantly, false signals are distinct from imprecise signals because they should be subsequently corrected. This possibility is typically ignored in extant theories about the effects of news arrivals and yet important empirically.

Our model considers the market for one risky asset (say a stock). Speculators receive a signal (a tweet, a rumor on an internet forum, news from “google news”, newswires, or news analytics from Reuters or Bloomberg, etc.) about the asset and trade with uninformed, but rational, market participants in two trading rounds. The signal received by speculators can be informative or just noise (false). Each signal is characterized by its *reliability*, i.e., the likelihood that it is not false. When they receive a signal, speculators cannot immediately tell its nature (false/true) without further analysis (“information processing”), which takes one period. Thus, speculators can follow two types of strategies: (a) trade on signals before and after processing them, at the risk of trading on noise or (b)

trade on signals only after processing them, at the risk of losing a profitable opportunity. Both strategies require speculators to pay a cost for receiving signals (e.g., a subscription fee to a news analytics' provider such as Thomson-Reuters). In addition, trading fast on information (i.e., before others process it) require investing in fast trading technologies (e.g., to pay exchange fees to co-locate speculators' algorithms close to exchanges' servers).

When the cost of trading fast is high enough, all speculators optimally choose to be slow, that is, to process signals before trading. Hence, they never trade on false information. As this cost falls, the number of fast speculators, who trade on signals without processing them, increases whereas the number of slow speculators declines. When the cost of trading fast becomes low enough, all speculators are fast: they trade on signals both before and after processing them.

This model has a rich set of testable implications. First, it predicts that, when the cost of trading fast decreases, the demand for information should decrease down to a point and then increase. A second implication is that speculators' order imbalances (i.e., their aggregate net trade) can be positively or negatively autocorrelated depending on (i) the reliability of their signals and (ii) the cost of trading fast. Finally, when the cost of trading fast declines, the likelihood of a price reversal after the first trading round increases because more speculators choose to trade on the signal without processing it.

1 Introduction

Improvement in trading technologies enable speculators to react to news in a few milliseconds.¹ However, in reacting fast to information, speculators take the risk of trading on false news as information processing (e.g., checking news accuracy) takes time. The “Twitter Crash” of April 2013 is one example. At 1:08pm on April 23rd, 2013 a fake tweet from a hacked Associated Press account announced that explosions at the White House had injured Barack Obama. As Figure 1 shows, the Dow Jones immediately lost 145 basis points but it recovered in less than three minutes after the news proved to be false.



Figure 1: The Twitter Crash: The Dow Jones Index on April 23, 2013.

The Twitter Crash is not an isolated example. Market participants claim that large sudden price drops or spikes followed by quick price reversals (i.e., “V-shape” or “inverted V-shape” price movements) are increasingly common and have nicknamed these patterns “mini flash crashes”. For instance, according to an article from the Huffington Post: “[...] *mini-flash crashes happen all of the time now. Just Monday, shares of Google collapsed briefly in a barely noticed flash crash of one of the country’s biggest and most important companies.*”² Similarly, Nanex (a financial data provider) reports more than 18,000 mini flash-crashes from 2006 to 2010 in U.S. equity markets, that is, about 195 per month

¹For instance, when the FED announced that it would not scale back on its buying program on September 18, 2013, stock prices immediately spiked in less than a microsecond. See “High speed traders reacted instantly to FED” on CNNMoney (<http://money.cnn.com/2013/09/19/investing/fed-high-speed-trading/>).

²See Huffington Post, “Twitter causes a flash crash, highlighting market’s structural problems,” 04/23/2.

(Nanex defines a flash-crash as an up or down price movement greater than 0.8% in less than 1.5 second).³

Mini-flash crashes are a source of concerns as they seem symptomatic of market fragility and informational inefficiency. In particular, sharp price drops in one asset might propagate to other assets leading to market-wide disruptions, as observed during the 2010 flash crash. Media, practitioners, and regulators link mini-flash crashes and more generally market instability to the growth of computerized and fast trading (the so called “race to zero”).⁴ For instance, in an article on the Twitter Crash, the Economist Magazine writes:⁵

“Twitter’s credibility (a novel idea to non-tweeters) has taken a hit. But human users must extract some sort of signal every day from the noise of innumerable tweets. Computerised trading algorithms that scan news stories for words like “explosions” may have proved less discerning and triggered the sell-off. That suggests a need for more sophisticated algorithms that look for multiple sources to confirm stories.”

However, academic studies do not support the view that computerized fast trading is associated with less efficient markets. For instance, using Nasdaq data, Brogaard, Hendershott, and Riordan (2013) conclude that high-frequency traders contribute to price discovery. Boehmer et al.(2013) reach a similar conclusion in a cross-country study.

This disconnect between anecdotal and academic evidence raise the following question: can faster trading on information make financial markets simultaneously more informationally efficient *and* unstable? Our goal in this paper is to study this question, building up on the idea that speculators might receive false signals and that processing signals (e.g., checking the validity of a signal by obtaining additional signals) takes time. Importantly, false signals are distinct from imprecise signals because they should be subsequently corrected. This possibility is typically ignored in extant theories about the effects of news arrivals (see below for a discussion) and yet important empirically.⁶

Our model considers the market for one risky asset (say a stock). Speculators receive

³See <http://www.nanex.net/FlashCrashEquities/FlashCrashAnalysisEquities.html>.

⁴See, for instance, the speech (“The race to zero”) of Andrew Haldane (Executive Director, Financial Stability, of the Bank of England) at the International Economic Association Sixteenth World Congress in Beijing, 2011. Available at: <http://www.bis.org/review/r110720a.pdf>.

⁵The Economist, “#newscrashrecover”, April 27, 2013.

⁶For instance, von Beschwitz, Keim, and Massa (2013) study the reaction of prices to news analytics provided by RavenPack. They find cases of news that were released as being highly relevant for a stock by RavenPack but that more recent versions of its textual-analysis software classify as having in fact low relevance. They show (see their Figure 4), that, for these news, prices follow a “V-shape” behavior: they initially increase or decrease (depending on the direction of the news) and then quickly revert (in less than 120 seconds). This suggests again that (i) news are sometimes false and (ii) participants learn gradually about the nature of news so that price reactions to false news are eventually corrected.

a signal (a tweet, a rumor on an internet forum, news from “google news”, newswires, or news analytics from Reuters or Bloomberg, etc.) about the asset and trade with uninformed, but rational, market participants (dealers, as in Glosten and Milgrom (1985)) in two trading rounds. The signal received by speculators can be informative or just noise (false). Each signal is characterized by its *reliability*, i.e., the likelihood that it is not false. When they receive a signal, speculators cannot immediately tell its nature (false/true) without further analysis (“information processing”), which takes one period. Thus, speculators can follow two types of strategies: (a) trade on signals before and after processing them, at the risk of trading on noise or (b) trade on signals only after processing them, at the risk of losing a profitable opportunity. Both strategies require speculators to pay a cost for receiving signals (e.g., a subscription fee to a news analytics’ provider such as Thomson-Reuters). In addition, trading fast on information (i.e., before others process it) require investing in fast trading technologies (e.g., to pay exchange fees to co-locate speculators’ algorithms close to exchanges’ servers).

When the cost of trading fast is high enough, all speculators optimally choose to be slow, that is, to process signals before trading. Hence, they never trade on false information. As this cost falls, the number of fast speculators, who trade on signals without processing them, increases whereas the number of slow speculators declines. When the cost of trading fast becomes low enough, all speculators are fast: they trade on signals both before and after processing them.

This model has a rich set of testable implications. First, it predicts that the demand for information should be a U-shape function of the cost of trading fast. On the one hand, a reduction in this cost makes the net expected profit of speculators who trade twice on their information (before and after processing it) larger. On the other hand, it reduces speculators’ expected profit when they trade on information after processing it. The former effect raises the value of information while the latter reduces it. This latter effect dominates only when the cost of trading fast is large enough so that not all speculators choose to trade fast on information. Hence, the demand for information can decrease or increase when the cost of trading fast is reduced.

A second implication is that speculators’ order imbalances (i.e., their aggregate net trade) can be positively or negatively autocorrelated depending on (i) the reliability of their signals and (ii) the cost of trading fast. To see why, suppose that speculators first receive a negative signal about a firm (e.g., “google news” announces that the firm is

bankrupt).⁷ In equilibrium, all speculators who choose to trade before processing the signal optimally sell the stock and if their selling pressure is strong enough, the stock price falls. If, after processing the signal, speculators realize that it is false, they all (including those who traded without initially processing the signal) place buy orders to exploit (and thereby correct) the erroneous initial price drop. If, in contrast, the signal is correct then all speculators sell additional shares and the initial price drop continues.

The first type of sequence ((a) signal, (b) trade, (c) signal correction) is a source of negative autocorrelation in speculators' order imbalances whereas the second type ((a) signal, (b) trade, (c) signal confirmation) is a source of positive autocorrelation in these imbalances. The former effect dominates (speculators' trades are negatively autocorrelated) iff the cost of trading fast on information is low enough and for *intermediate* values of signals' reliability. Indeed, trade reversals happen when (a) signals turn out to be false positive (which is more likely when signals are less reliable) **and** (b) fast speculators' trades move prices, which is more likely if their mass is large enough. This happens when the cost of trading fast on information is low and information is sufficiently reliable.

Another implication is that, depending on signal reliability, speculators may behave as contrarian or momentum traders. If the reliability of speculators' signals is small, speculators behave as contrarian traders: the direction of their trade *after* processing information is *negatively* correlated with the return following news arrival. Indeed, when signal reliability is low, price movements following the arrival of speculators' signals are more likely to be due to false news and thereby to be subsequently corrected by trades in the opposite direction of the initial price movement. In contrast, when signals' reliability is high, speculators' signals are more likely to be confirmed subsequently (false news are rare), which trigger additional speculators' trades in the same direction as the price movement following news arrivals. In this case, speculators behave like momentum traders: their trades after processing information are positively correlated with past returns. Thus, the effect of past returns on high-frequency (fast) traders' net order imbalances should depend on the reliability of signals received by these traders.⁸

⁷On Monday 8, 2008, the stock price of United Airlines dropped to \$3 a share from nearly \$12 in about fifteen minutes. Then the price bounced back at \$11 at the end of the Tuesday session. The cause of this price swing was an old article about United Airlines' 2002 bankruptcy-court filing that mistakenly appeared on September 8, 2008 as a seemingly new headline on Google's news service.

⁸One could test this prediction using VAR analysis by allowing lagged returns to affect fast traders' order imbalances (as in Hirschey (2013)) and checking whether the sign of coefficients on lagged returns depends on a measure of news reliability. Brogaard et al.(2013) (see their Figure 2) find that net trades (buys minus sales) of high frequency traders are negatively correlated with past returns at the high frequency. Hirschey (2013) find that high frequency traders' order imbalances are positively related to the first three lagged returns (measured at the one second frequency) and then negatively related to

When the cost of trading fast declines, the likelihood of a price reversal after the first trading round increases because more speculators choose to trade on the signal without processing it.⁹ The likelihood of a price reversal is an inverse U-shape function of signal reliability. The reason is that the number of speculators who trade on a signal without first processing it increases with signal reliability. Thus, conditional on the signal being false, the likelihood of an erroneous price movement following news arrival, and therefore a subsequent reversal, is larger for more reliable news. The likelihood that the signal is false is smaller, however so that eventually the relationship between the frequency of price reversals and news reliability is non-monotonic.

We define a mini-flash as a price change in the first trading round that (i) is larger than a fixed threshold R (say $x\%$ of the range of the payoff distribution for the asset) and (ii) reverts in the second period. We show that, for any value of R , mini-flash crashes become more frequent when the cost of trading fast decreases, as observed in recent years. In addition, the frequency of mini-flash crashes in a stock is an inverse U-shape function of the average news reliability in this stock.

Finally, we study the effect of the cost of trading fast on informational efficiency, measured by the mean-squared pricing error (i.e., the squared difference between the asset price and the asset payoff) in each trading round. Not surprisingly, a decline in the cost of trading fast makes the price in the first period more informationally efficient: signals contain information even though they are unreliable and having more speculators trading fast accelerate price discovery. More interestingly, informational efficiency *after* speculators process information is either unchanged or even improved when the cost of trading fast declines. This finding is surprising because, as mentioned previously, a decline in the cost of trading fast can reduce the number of informed investors and always increases the number of speculators who take the risk of trading on false signals. One might expect these effects to harm efficiency.

However, there is a countervailing force: gradual trading on signals enable dealers to better filter out information from order flows. Indeed, dealers face two sources of uncertainty: (i) they do not know the direction of speculators' signal and (ii) they do not know whether speculators' signal is valid or not. Gradual trading by speculators,

returns at higher lags (see Table 4 in Hirschey (2013)). However there is cross-sectional variations in the signs of these coefficients. Our model suggests news reliability as a potential determinant of this cross-sectional variability.

⁹Prices are semi-strong form efficient in our model. Hence, consecutive price changes are uncorrelated. However, conditional on the signal being false, a price change in the first period revert in the second period.

first on the direction of the signal and then on its validity, help dealers to better learn about these two dimensions of uncertainty. Thus, paradoxically, a reduction in the cost of trading fast can simultaneously increase the frequency of mini-flash crashes and improve price discovery.

As in Froot, Scharfstein and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994), and Brunnermeier (2005), we consider a situation in which some informed traders can trade on their signal before other informed traders. Our model differs from these models in at least three important ways: (i) speculators receive signals that might be false, (ii) they *gradually* learn whether signals are valid or false, and (iii) speculators observe their signal simultaneously but *endogenously* choose to trade early or not on the signal. These features are absent from models with early and late informed traders and they are key for several of our predictions. For instance, our predictions regarding the effects of the cost of trading fast stem from the possibility for speculators to choose whether to trade early (before processing information) or not.¹⁰

As far as we know, our paper is first to analyze a model of trading with false news. There exist many static (one trading round) models in which informed investors receive imprecise signals (e.g., Kim and Verrechia (1994)). In static models, however, there is no difference between imprecise signals and false signals as speculators cannot separate information from noise. However, if signals can be false, in a dynamic setting, it is natural to consider the possibility that speculators receive additional signals correcting earlier signals (i.e., learn about the noise in their initial signal). Accounting for this could be important in analyzing the effects of news on stock prices. For instance, using textual analysis, Boudoukh et al.(2013) identify days with no or unidentified news and days with news. They find that stock returns revert on the former and exhibit small continuations on the latter. Our model predicts exactly this pattern if days with no or unidentified news happen to be days with false news (and maybe classified for this reason as days with no or unidentified news by textual analysis).

Several papers consider cases in which informed investors receive multiple signals over time (e.g., Back and Pedersen (1998), Chau and Vayanos (2008), or Foucault, Hombert, and Rosu (2012)). In these models, informed investors' signals can be imprecise. However,

¹⁰This possibility is also key for our results regarding price reversals and mini-flash crashes because the likelihood of an erroneous price movement is positively related to the mass of speculators who choose to trade before processing information. In Froot et al.(1992), there exist equilibria in which a fraction of speculators trade on noise. However, there is no possibility for traders to correct price changes due to such trades. In contrast, in our model, speculators correct erroneous price changes in period 2, after processing signals.

informed investors do not receive additional signals about the noise in early signals. That is, early signals are not subsequently corrected. To our knowledge, our paper is first to introduce this possibility in models of informed trading. Biais, Foucault, and Moinas (2012) and Jovanovic and Menkveld (2012) develop models in which fast traders' advantage stems from a quick access information. However, in these models, informed investors process news instantaneously. Hence, they face no trade-off between trading fast on very noisy information or waiting to obtain a more precise signal.

The next section describes the model and Section ?? derives equilibrium prices at dates 1 and 2, taking the number of fast and slow speculators as given. Section ?? endogenizes the number of speculators of each type and shows that the demand for information is a U-shape function of the cost of being fast. Section ?? considers the implications of the model for (a) price and trade patterns, (b) informational efficiency, and (c) the frequency of mini-flash crashes. Section ?? concludes.

2 Model

We consider a model of trading in the market for a risky asset with four periods ($t \in \{0, 1, 2, 3\}$). The payoff of the asset, V , is realized at date $t = 3$. It can be $V = 1$ or $V = 0$ with equal probabilities. Trades take place at dates $t = 1$ and $t = 2$. There are three types of market participants: (i) liquidity traders, (ii) a continuum of speculators, (iii) a competitive market-maker. Figure ?? summarizes the timing of actions and events in our model.

We now describe in more detail traders' actions at dates 0, 1, and 2.

Liquidity Traders. As in Glosten and Milgrom (1985), liquidity traders need to buy or sell the asset for exogenous reasons. Their aggregate demand at date $t \in \{1, 2\}$ is denoted \tilde{l}_t . It is uniformly distributed over $[-Q, Q]$ with density:

$$\phi(x) = \frac{1}{2Q} \times \mathbb{I}_{\{x \in [-Q, Q]\}}. \quad (1)$$

Fast and Slow Speculators. At date 0, speculators make two decisions: (i) they decide to buy or not information at cost C_p and (ii) to invest or not in a fast trading technology at cost Δ . If an investor acquires information, he receive a signal $\tilde{S}_1 \in \{0, 1\}$

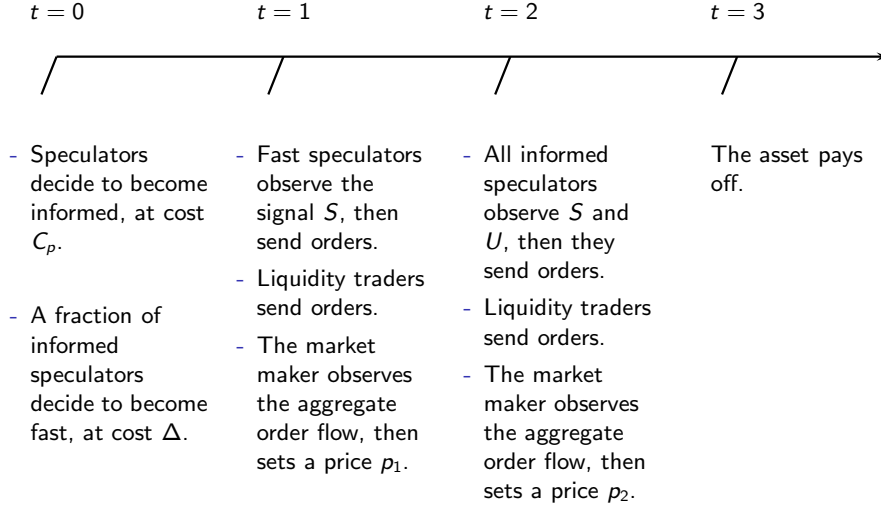


Figure 2: Market Participants' Decisions: Timing.

about the payoff of the asset just before the first trading round at $t = 1$, with:

$$\tilde{S}_1 = \tilde{U} \times \tilde{V} + (1 - \tilde{U}) \times \tilde{\epsilon}, \quad (2)$$

where $\tilde{U} \in \{0, 1\}$ and $\tilde{\epsilon} \in \{0, 1\}$. Moreover, $Pr[U = 0] = 1 - \theta$ and $Pr[U = 1] = \theta$, with $Pr[\epsilon = 0] = Pr[\epsilon = 1] = 1/2$. Thus, with probability θ , the signal is informative and with probability $(1 - \theta)$, the signal is just noise. Hence, θ measures the *reliability* of the signal received by speculators at date 0. Speculators can process the signal to determine whether it is informative ($U = 1$) or not ($U = 0$). Information processing however takes time. Hence, U cannot be discovered before $t = 2$. A speculator can trade on the signal *before* processing it, at date $t = 1$, only if he owns the fast trading technology.

We refer to speculators who invest in the fast trading technology as *fast speculators* and to speculators who only trade at date $t = 2$ as *slow speculators*. The total cost borne at date $t = 0$ by fast speculators is $C_F = C_p + \Delta$ whereas slow speculators bear a smaller cost, C_p . We denote by β the mass of speculators (fast and slow) who acquire information and by α the mass of fast speculators. We have $\alpha \leq \beta$ because fast speculation requires buying information in the first place.¹¹

¹¹A speculator without information expects a zero profit in our model because he expects to buy or sell the asset at its fair value. Thus, investing in the fast trading technology without buying information

Costs C_p and Δ are of different nature. Cost C_p is an information acquisition cost. It represents, for instance, the cost that speculators pay to obtain news analytics services (e.g., from Bloomberg, Reuters etc.) and to process information (skilled analysts, computers, etc.). In contrast, cost Δ is a technological cost that is paid to trade fast once the information is received. This cost represents investments made by high frequency trading firms to maximize their speed of reaction to market events. One proxy for this cost could be, for instance, the fee charged by exchanges for colocation services.¹² Hence, β can be seen as the demand for information and α as the demand for fast trading technologies by investors buying information. We endogenize α and β in Section ??.

Order Flow. Let p_t be the stock price at date t . As in Glosten and Milgrom (1985), speculators only place market orders (i.e., orders that are non contingent on execution price) of a fixed size, which is normalized to one share. This assumption is not key because we do not a priori restrict α and β , the masses of speculators trading at dates 1 and 2. Hence, the total number of shares purchased or sold by speculators at each date can be large, even though the trade of each speculator is small.

We denote the market order of speculator i at dates 1 and 2 by $x_{i1}(s)$ and $x_{i2}(s, u, p_1)$, respectively, where s and u are the realizations of S_1 and U and $x_{it} \in \{-1, 0, 1\}$. A market order specifies the number of shares purchased or sold by the speculator given his information ($x_{it} < 0$ means selling $|x_{i1}(s)|$ shares at date t). At date 2, a speculator's market order depends on his information (s and u) and the last transaction price, p_1 . At date 1, $x_{i1}(s) = 0$ for speculators who do not have the fast trading technology.

The aggregate order flow at date t , f_t , is therefore:

$$f_t = \tilde{l}_t + \int_0^\beta x_{it}(s) di. \quad (3)$$

We denote by f_t^{max} and f_t^{min} , the highest and smallest possible values of the order flow at date t . Obviously, $f_1^{max} = Q + \alpha$, $f_1^{min} = -Q - \alpha$, $f_2^{max} = Q + \beta$, and $f_2^{min} = -Q - \beta$.

The Market-Maker At each date, the market-maker sets a price, p_t , for the asset equal to the expected payoff of the asset conditional on his information. We assume that the market-maker does not observe \tilde{S} and \tilde{U} until date $t = 3$ because our goal is to model

 results in a loss equal to the investment Δ . Hence, no speculator would invest in the technology without acquiring information.

¹²Colocation enables proprietary traders to locate their computers very close to exchanges' servers. In this way, traders reduce the time it takes for them to send new orders (e.g., in reaction to a signal arrival) to exchanges.

price dynamics just after the arrival of information, before information become known by all market participants. The market-maker's quotes however are contingent on the order flow, f_t , in each period (as in Kyle (1985)).

Formally, let Ω_t be the information set of the market maker at date t ($\Omega_1 = f_1$ and $\Omega_2 = \tilde{f}_2, \tilde{f}_1$). The stock price at date t is:

$$p_t = E[V|\Omega_t] = Pr[V = 1|\Omega_t]. \quad (4)$$

A Proxy for Signal Reliability. In our model, speculators first observe a noisy signal, S_1 , about V . Then, at date $t = 2$, they learn whether this signal was false or informative. This information structure captures the idea that information processing takes time and consists in filtering out noise from signals. Another, equivalent approach, to formalize the same idea is to assume that speculators receive a sequence of signals about V . For instance, suppose that at date 2, instead of observing U directly, speculators receive a signal S'_2 such that:

$$S'_2 = S_1, \quad \text{if } U = 1,$$

and

$$S'_2 = \frac{1}{2}, \quad \text{if } U = 0.$$

This information structure captures the idea that a false signal at date 1 is subsequently corrected while a valid signal is not. Let ρ_θ be the autocorrelation between the signal at date 1 and the change in the signal from date 1 to date 2, $(S'_2 - S_1)$. Calculations yield:

$$\rho_\theta = -\sqrt{(1 - \theta)}.$$

Thus, the possibility for the signal at date 1 to be false ($\theta < 1$) and therefore corrected in the future (here at date 2) implies that changes in signals are negatively correlated. Moreover, the smaller is a signal reliability (θ), the stronger is the negative autocorrelation in signals. Hence, empirically the autocorrelation in the innovations of signals received by market participants could be used to detect whether these signals are sometimes false (in this case the autocorrelation in signals should be negative). Furthermore the inverse of the absolute autocorrelation of these innovations can be used as a proxy for reliability.¹³

¹³For instance, news analytics providers assign a sentiment score and relevance score to each news (e.g., +1 if the news is interpreted as positive by news analytics software and -1 if it is interpreted as negative. One could use innovations in these scores to measure the autocorrelation in news for the same stock. If less relevant news are also less reliable, the autocorrelation in news might become negative for

3 Equilibrium Trading Strategies and Prices

In this section, we derive speculators' optimal trading strategies and equilibrium prices, taking the demands for information and fast trading technologies, β and α , as given. Let $\mu(s)$ be the expected payoff of the asset at date 1 when the realization of the signal observed by speculators at date 1 is $s \in \{0, 1\}$. We have:

$$\mu(s) = Pr[V = 1|S_1 = s] = \frac{Pr[S_1 = s|V = 1]Pr[V = 1]}{Pr[S_1 = s]}.$$

Hence:

$$\mu(1) = \frac{1 + \theta}{2} > \frac{1}{2} \quad \text{and} \quad \mu(0) = \frac{1 - \theta}{2} < \frac{1}{2}.$$

The expected profit of a speculator who trades x_1 shares in period 1, is therefore

$$\pi_1(\alpha, s) = x_1(\mu(s) - E[p_1|S_1 = s]).$$

We denote the ex-ante expectation of this profit by $\bar{\pi}_1(\alpha)$. The next proposition describes the equilibrium (prices and fast speculators' trading strategies) at date 1 and provides the equilibrium ex-ante expected profit for the fast speculators on their first period trade.

Proposition 1. *The equilibrium at date 1 is such that:*

1. *A fast speculator buys one share if his signal is high and sells one share if his signal is low:*

$$x_1(1) = 1, \quad x_1(0) = -1. \quad (5)$$

2. *The equilibrium stock price at date 1 is:*

$$p_1(f_1) = Pr[V = 1|\tilde{f}_1 = f_1] = \frac{1}{2} \left(\frac{(1 + \theta)\phi(f_1 - \alpha) + (1 - \theta)\phi(f_1 + \alpha)}{\phi(f_1 - \alpha) + \phi(f_1 + \alpha)} \right), \quad (6)$$

for $f_1 \in [f_1^{min}, f_1^{max}]$.

3. *Fast speculators' expected profit at date 1 is:*

$$\bar{\pi}_1(\alpha) = \frac{\theta}{2} \text{Max}\left\{\frac{Q - \alpha}{Q}, 0\right\}. \quad (7)$$

As expected, fast speculators buy when they observe a good signal and sell when they receive a bad signal. Thus, the order flow at date 1 is positively correlated with the news with a relevance sufficiently low.

signal received by speculators and is therefore informative about this signal. This signal is noisy however because the order flow is also determined by liquidity traders' orders, which contain no information. For these reasons, the price of the asset at date 1 weakly increases in the order flow. Specifically, using (??), we deduce from (??) that:

$$p_1(f_1) = \begin{cases} \mu(0) & \text{if } f_1 \in [f_1^{min}, -Q + \alpha], \\ \frac{1}{2} & \text{if } f_1 \in [-Q + \alpha, Q - \alpha], \\ \mu(1) & \text{if } f_1 \in [Q - \alpha, f_1^{max}]. \end{cases}$$

When the order flow at date 1 belongs to $[-Q + \alpha, Q - \alpha]$, the market-maker cannot infer speculators' signal because realizations of order flow in this range are equally likely when $V = 1$ and $V = 0$. Thus, the market-maker sets a price equal to the ex-ante expected value of the asset. If the buying pressure is strong enough ($f_1 \geq Q - \alpha$), the market-maker infers that speculators are buying and deduces that $S_1 = 1$. If instead the selling pressure is strong ($f_1 \leq -Q + \alpha$), the market-maker infers that speculators are selling and deduces that $S_1 = -1$. When α increases, the range of values for the order flow such that it fully reveals speculators' signal gets larger because speculators account for a larger fraction of the trading volume. Correspondingly, their expected profit in the first period declines because speculators can make a profit on the first round signal only if prices do not fully reflect their information.

If $Q \leq \alpha$, the mass of fast speculators is so large relative to the mass of liquidity traders that the order flow is always fully revealing (the interval $[-Q + \alpha, Q - \alpha]$ is empty). In this case, fast speculators make a zero expected profit at $t = 1$.¹⁴

At $t = 2$, speculators observe the realization of \tilde{U} : they learn if the first period signal is false or not. Hence, at $t = 2$, the expected profit of a speculator is:

$$\pi_2(\alpha, \beta, s, u) = x_2(E[V|U = u, S_1 = s] - E[p_2|U = u, S_1 = s, p_1]).$$

We denote by $\bar{\pi}_2(\alpha, \beta)$, the ex-ante (i.e., date 0) speculators' expected profit on their trade at date 2. This expected profit does not depend on whether the speculator is fast or not because, at date 2, all speculators have identical information and therefore follow

¹⁴When $Q \leq \alpha$, the equilibrium described in Proposition ?? is unique when $\alpha < Q$ and it is the unique equilibrium in symmetric pure strategies when $\alpha \geq Q$. When $\alpha \geq Q$, speculators make zero expected profit on all orders. Hence, there are also mixed strategy equilibria in this case. For instance, one can construct equilibria in which only a mass Q of speculators trade. However, in all equilibria, speculators expect a zero profit. Furthermore, the equilibrium mass of fast speculators is strictly less than Q when $\Delta > 0$ (see Section ??). Thus, $\alpha < Q$ is the more relevant case.

the same strategy. The next proposition provides the equilibrium at date $t = 2$ and the ex-ante expected profit of speculators on their trades at this date.

Proposition 2. *Let $M_0(p_1) = 2\beta(\frac{1}{2} - p_1)$. The equilibrium at date $t = 2$ is such that:*

1. *If signal S_1 is informative ($U = 1$), speculators buy one share if $tS_1 = 1$ ($x_2(1, 1, p_1) = 1$) and sell one share if $S_1 = 0$ ($x_2(-1, 1, p_1) = -1$). If signal S_1 is false ($U = 0$), speculators buy one share if the price in the first period is less than $\frac{1}{2}$ ($x_2(s, 0, \mu(-1)) = -1$), sell one share if the price in the first period is greater than $\frac{1}{2}$ ($x_2(s, 0, \mu(1)) = 1$), and do not trade otherwise ($x_2(s, 0, \mu(0)) = 0$).*
2. *Hence, speculators' aggregate demand at date $t = 2$ is $M_0(p_1) = 2\beta(\frac{1}{2} - p_1)$ when $U = 0$ and $M_1(S_1) = 2\beta(S_1 - \frac{1}{2})$ when $U = 1$.*
3. *The equilibrium stock price at date 2 is*

$$p_2(f_2, f_1) = \frac{\theta\phi(f_1 - \alpha)\phi(f_2 - \beta) + \frac{1-\theta}{2}[\phi(f_1 - \alpha) + \phi(f_1 + \alpha)]\phi(f_2 - M_0(p_1))}{\theta[\phi(f_1 - \alpha)\phi(f_2 - \beta) + \phi(f_1 + \alpha)\phi(f_2 + \beta)] + (1 - \theta)[\phi(f_1 - \alpha) + \phi(f_1 + \alpha)]\phi(f_2 - M_0(p_1))}. \quad (8)$$

for $f_2 \in \{f_2^{min}, f_2^{max}\}$.

4. *Speculators' ex-ante expected profit at date 2 is continuous in α and β . Furthermore:*

$$\bar{\pi}_2(\alpha, \beta) = \begin{cases} \frac{\theta}{2Q^2} \times [(Q - \alpha) \times (Q - \beta(2 - \theta)^{-1}) + \alpha(1 - \theta)(Q - \beta)] & \text{if } \beta \leq Q, \\ \frac{\theta}{2Q^2} (\frac{1-\theta}{2-\theta})(Q - \alpha)(2Q - \beta) & \text{if } Q \leq \beta \leq 2Q, \\ 0 & \text{if } \beta > 2Q. \end{cases} \quad (9)$$

The market-maker's valuation for the asset after observing trades at date 1 is p_1 . At date 2, speculators buy the asset if their expectation of its payoff is higher than the market-maker's valuation and sells it otherwise, exactly as in period 1. As a result, they trade in the same direction as in the first period if the signal is indeed informative ($Cov(M_1(S_1 - 1), S_1) > 0$ if $U=1$). If instead, the signal is false and the price reacted in the first period, speculators trade in the opposite direction of the first period return to correct it ($Cov(M_1(S_1 - 1), p_1) < 0$ if $U=0$). Hence, in the first case, speculators follow a momentum strategy (they trade in the same direction as the first period return) while

in the second case they follow a contrarian strategy (they trade in a direction opposite to the first period return). Fast speculators trade in both periods. Thus, when the signal is false, they unwind the position acquired in the first period. Otherwise they accumulate more shares. In Section ??, we study more systematically the implications of the model for time-series patterns in returns and speculators' trades.

As in period 1, the order flow is informative because speculators buy (sell) the asset when the last period price is too low (high) compared to their forecast of the asset payoff. Thus, the second period price is again weakly increasing in the second period order flow. Using the characterization of equilibrium price at dates 1 and 2, Figure ?? shows the equilibrium price dynamics conditional on $S_1 = 1$ and $S_1 = 0$, for fixed value of α and β (less than Q) and for all possible realizations of U at date 2. The probability of each possible price change (up, down, or no change) at dates 1 and 2 are shown on each branch. The unconditional probability of a given price path in equilibrium is obtained by multiplying the likelihood of this path by $1/2$.

For instance, suppose that $S_1 = 1$. In this case, all fast speculators buy in period 1 and with probability $\frac{\alpha}{Q}$, this buying pressure is strong enough to push the price up. At date 2, with probability θ , speculators learn that the signal in period 1 was indeed correct. Hence, they keep buying the asset and with probability $\frac{\beta}{Q}$, the buying pressure at date 2 is so strong that the market maker infers that $V = 1$. Hence, the price goes up as well at date 2. The overall unconditional probability of two consecutive up movements in the price is therefore $\frac{1}{2} \frac{\alpha\theta\beta}{Q^2}$.

Alternatively, with probability $(1-\theta)$, speculators learn in period 2 that the first period signal is false. Hence, they revise their initial expectation about the asset payoff from $\mu(1) = 1$ to $E(V) = 1/2$. If the price has not changed in period 1, they are indifferent between trading or not at date 2 because they expect to trade the asset at $1/2$.¹⁵ Thus, they choose not to trade ($M_0(\frac{1}{2}) = 0$). If instead the price has increased in the first period, speculators sell the asset in period 2 and with probability $\frac{\beta}{Q}$, the selling pressure is strong enough to push the price back to its initial level. Thus, the unconditional probability of an up price movement followed by a down movement is $\frac{1}{2} \frac{(1-\theta)\alpha\beta}{Q^2}$.

Interestingly, the sequence of trades and price movements following false news might be interpreted by outside observers (e.g., regulators) as an attempt to manipulate the market by speculators. Indeed, speculators first buy the asset, the price increases as a

¹⁵The reason is that a speculator expects (i) liquidity traders' aggregate demand and (ii) each speculator' demand to be zero. Hence, a speculator expects the price at date 2 to be identical to the price at date 1 price because his demand is negligible compared to speculators' aggregate demand.

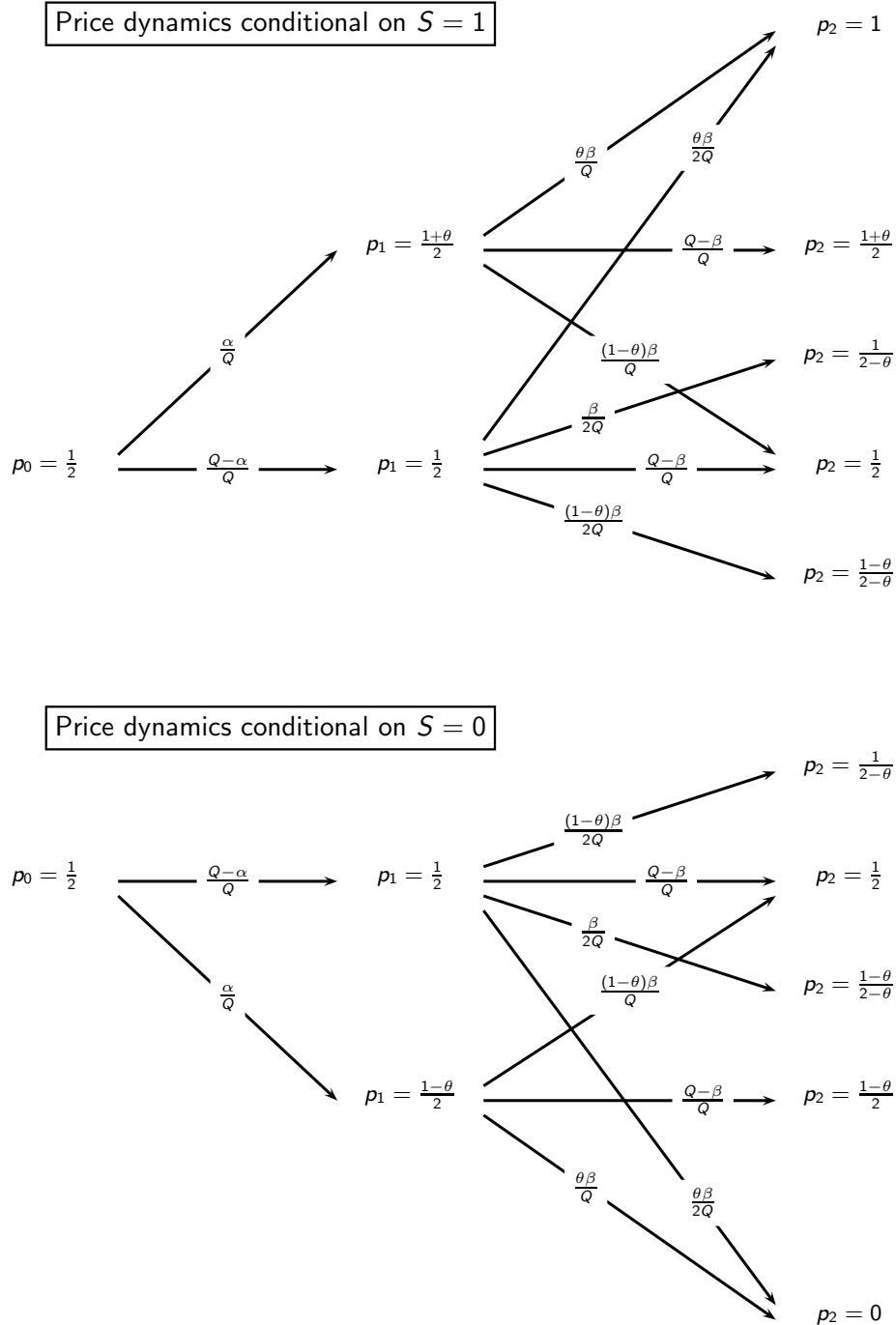


Figure 3: Price dynamics in equilibrium.

result, and finally speculators turn around their position to correct the erroneous price increase. This sequence of events has the flavor of so called *momentum ignition strategies* described by the SEC as follows:

“With this strategy, the proprietary firm may initiate a series of orders and trades (along with perhaps spreading false rumors in the marketplace) in an attempt to ignite a

rapid price move either up or down. For example, the trader may intend that the rapid submission and cancellation of many orders, along with the execution of some trades, will "spoo" the algorithms of other traders into action and cause them to buy (sell) more aggressively. [...] By establishing a position early, the proprietary firm will attempt to profit by subsequently liquidating the position if successful in igniting a price movement." (SEC, concept release on Equity market structure (2010)).

If, as is likely in reality, speculators do not all move simultaneously, but rather sequentially, one will in fact have in fact the impression that early traders are igniting the trades of other traders. Furthermore, conditional on false news (which might well be perceived as "rumors"), early traders liquidate their position in the second period if the price moved in the first period (i.e., "if successful in igniting a price movement"). Yet, there is no price manipulation in our model. In equilibrium, speculators' behavior is just a consequence of the fact that they might optimally choose to trade before processing news and news being false.

The market maker's expected profit at date 2 is positive as long as $\beta \leq 2Q$. As explained in the next section, it will be satisfied in equilibrium when the cost of acquiring information is large enough.

Corollary 1. *In equilibrium, for fixed values of α and β :*

1. *Speculators' ex-ante expected profit at date 1 increases with θ , the reliability of information,*
2. *If $\beta \leq Q$ and $\frac{\alpha}{Q-\alpha} + \frac{\beta}{Q-\beta} < 1$, speculators' ex-ante expected profit at date 2 increases with θ .*
3. *If $\beta \leq Q$ and $\frac{\alpha}{Q-\alpha} + \frac{\beta}{Q-\beta} > 1$, there is unique $\Theta(\alpha, \beta) \in [0, 1]$ (defined in the proof) such that speculators' ex-ante expected profit at date 2 increases with θ , on the interval $[0, \Theta(\alpha, \beta)]$, and decreases with θ on the interval $[\Theta(\alpha, \beta), 1]$.*
4. *If $Q < \beta \leq 2Q$, speculators' ex-ante expected profit at date 2 increases with θ iff $\theta \leq 2 - \sqrt{2}$.*

Furthermore, for a fixed value of θ , speculators' ex-ante expected profits at dates 1 and 2 decrease with α and β .

Fast speculators are less at risk of trading on a false information when the signal is more reliable. Thus, not surprisingly, the expected profit of fast speculators at date

1 increases with the reliability of their information. In contrast, the effect of signal reliability on speculators' expected profit at date 2 is less clear-cut. On the one hand, an increase in the signal reliability increases the likelihood for a speculator to be informed at date 2, which everything else equal enhances his expected profit. On the other hand, it raises the likelihood of a price movement in period 1. As θ increases, this price movement is more likely to contain information, which reduces the profit of informed trading at date 2 if the signal at date 1 is indeed informative. The latter effect is small when α and β are small or when θ is small enough. When α and β are large and when θ is large enough then the former effect dominates and speculators' expected profit decreases with θ .

The order flow at date 2 is more likely to be fully informative as the mass of speculators, β , increases. For this reason, speculators' ex-ante expected profit in period 2 decreases with β . Speculators' expected profit in period 1 decreases with α because an increase in the mass of fast speculators increases the likelihood that the order flow is fully informative. Speculators' expected profit at date 2 also depends on the mass of fast speculators, α for two reasons. First, an increase in α increases the likelihood that the price at date 1 adjusts in the direction of speculators' signal, S_1 . If the signal is correct ($U = 1$), an early adjustment of the price to the signal lowers the expected profit of trading "late" on signal S_1 . If instead the signal is false, this early adjustment is a source of profit for all speculators because they know that the first period price was erroneous. The second part of Corollary shows that the first effect always dominates, so that speculators' expected profit in period 2 decreases with the mass of fast speculators, α .

4 Demand for Information and the Cost of Fast Trading

We now derive the equilibrium demand for information and fast trading technologies, namely equilibrium masses, β^* and α^* , of speculators and fast speculators. Let $\beta^*(\alpha, C_p)$ be the equilibrium mass of speculators when the mass of fast speculators is α and let $\alpha^*(\Delta)$ be the equilibrium mass of fast speculators when the cost of trading fast is Δ . In any case $\beta^*(\alpha) \geq \alpha$ because fast speculators also process the signal and finally discovers whether the latter is informative or not. When $\beta^*(\alpha^*) > \alpha^* > 0$, fast and slow speculators coexist. When $\beta^*(\alpha^*) = \alpha^* > 0$, all speculators are fast whereas when $\alpha^* = 0$, all speculators are slow. We now analyze under which conditions each of these cases is obtained. In particular we study the effect of the cost of being fast, Δ , on traders' incentives to buy

information.

A speculator who only trades after processing information obtains a net expected profit of

$$\Pi^S(\alpha, \beta) = \bar{\pi}_2(\alpha, \beta) - C_p, \quad (10)$$

whereas a fast speculator obtains a total net expected profit of:

$$\Pi^F(\alpha, \beta) = \bar{\pi}_1(\alpha) + \bar{\pi}_2(\alpha, \beta) - (\Delta + C_p). \quad (11)$$

We now show that the demand for information, $\beta^*(\alpha^*)$, is a U-shape function of the cost of trading fast, Δ . For given α and β , the marginal value of the fast trading technology is:

$$\Pi^F(\alpha, \beta) - \Pi^S(\alpha, \beta) = \pi_1(\alpha) - \Delta. \quad (12)$$

As $\pi_1(\alpha)$ decreases with α , we deduce that no speculator chooses to be fast if:

$$\pi_1(0) \leq \Delta, \quad (13)$$

that is,

$$\Delta \geq \frac{\theta}{2}. \quad (14)$$

In this case, no speculator chooses to trade on the signal without first processing it. Thus, the equilibrium mass of speculators, solves:

$$\Pi^S(0, \beta^*(0)) = 0. \quad (15)$$

That is,

$$\bar{\pi}_2(0, \beta^*(0)) = C_p. \quad (16)$$

We deduce from the expression for $\bar{\pi}_2(., .)$ in Proposition ?? that:

$$\beta^*(0) = \begin{cases} 0 & \text{if } C_p > \frac{\theta}{2}, \\ (2 - \theta)Q \left(1 - 2\frac{C_p}{\theta}\right) & \text{if } \frac{\theta}{2} \left(\frac{1-\theta}{2-\theta}\right) < C_p < \frac{\theta}{2}, \\ 2Q \left(1 - (2 - \theta)\frac{C_p}{\theta(1-\theta)}\right) & \text{if } C_p \leq \frac{\theta}{2} \left(\frac{1-\theta}{2-\theta}\right). \end{cases} \quad (17)$$

Thus, $\beta^*(0)$ is greater than zero for $C_p < \frac{\theta}{2}$ and always less than $2Q$ for $C_p > 0$. Moreover, for $\frac{\theta}{2} \left(\frac{1-\theta}{2-\theta}\right) < C_p < \frac{\theta}{2}$, $\beta^*(0) \leq Q$ and for $C_p \leq \frac{\theta}{2} \left(\frac{1-\theta}{2-\theta}\right)$, $\beta^*(0) \geq Q$.

Now suppose that $\Delta < \frac{\theta}{2}$. In this case, some or all speculators choose to be fast:

$\alpha^* > 0$. Furthermore, if $\alpha^* < \beta^*(\alpha^*)$, some speculators choose to remain slow and the total mass of speculators adjusts so that the expected profit of just processing information is zero. That is, if $\alpha^* < \beta^*(\alpha^*)$:

$$\Pi^S(\alpha^*, \beta^*(\alpha^*)) = 0. \quad (18)$$

Consequently, in this case, the mass of fast speculators, α^* , must be such that:

$$\Pi^F(\alpha^*, \beta^*) = \bar{\pi}_1(\alpha^*) + \bar{\pi}_2(\alpha^*, \beta^*) - (\Delta + C_p) = \bar{\pi}_1(\alpha^*) - \Delta = 0,$$

so that in equilibrium, the marginal benefit of being fast ($\pi_1(\alpha^*)$) is just equal to the marginal cost (Δ). Hence, when both fast and slow speculators coexist:

$$\alpha^*(\Delta) = Q \left(1 - 2 \frac{\Delta}{\theta} \right). \quad (19)$$

As Δ decreases (starting from $\frac{\theta}{2}$), the mass of fast speculators increases in equilibrium because $\Pi^F(\alpha, \beta)$ decreases with α and Δ (see (??)). Accordingly, the expected profit of slow speculators decline (Proposition ??). Hence, the value of $\beta^*(\alpha^*)$ such that (??) holds is smaller because $\Pi^S(\alpha, \beta)$ decreases with β . Thus, there exists a threshold $\bar{\Delta}(\theta, C_p)$ such that $\beta^*(\alpha^*(\bar{\Delta})) = \alpha^*(\bar{\Delta})$, where $\alpha^*(\Delta)$ is given by eq.(??). This threshold solves:

$$\Pi^S(\alpha^*(\bar{\Delta}), \alpha^*(\bar{\Delta})) = \bar{\pi}_2(\alpha^*(\bar{\Delta}), \alpha^*(\bar{\Delta})) - C_p = 0. \quad (20)$$

Using the expression for $\bar{\pi}_2(\alpha, \beta)$ in Proposition ??, we have $\bar{\pi}_2(0, 0) = \frac{\theta}{2}$, $\bar{\pi}_2(Q, Q) = 0$, and $\bar{\pi}_2(\alpha, \alpha) \geq 0$, $\forall \alpha \in [0, Q]$. Hence, as $\bar{\pi}_2(\alpha, \beta)$ decreases in both α and β , we deduce that eq. (??) has always a unique positive solution, $\bar{\Delta}$. We provide a closed-form expression for $\bar{\Delta}(\theta, C_p)$ in the proof of Proposition ??.

Now, suppose that $\Delta < \bar{\Delta}$. We have $\alpha^*(\Delta) > \alpha^*(\bar{\Delta})$ because α^* decrease with Δ . Hence, as $\bar{\pi}_2(\alpha, \beta)$ decreases with α and β , it must be the case that:

$$\bar{\pi}_2(\alpha^*, \beta) - C_p < \bar{\pi}_2(\alpha^*, \alpha^*) - C_p < \bar{\pi}_2(\alpha^*(\bar{\Delta}), \alpha^*(\bar{\Delta})) - C_p = 0, \quad (21)$$

for $\beta \geq \alpha^*$. Thus, it cannot be optimal for a speculator to be slow when $\Delta < \bar{\Delta}$ and therefore in this case $\beta^*(\alpha^*) = \alpha^*$. In equilibrium, the mass of speculators must therefore adjust so that all speculators make zero expected on average (rather than trade by trade).

Thus, when $\Delta < \bar{\Delta}$, α^* solves:

$$\Pi^F(\alpha^*, \alpha^*) = \bar{\pi}_1(\alpha^*) + \bar{\pi}_2(\alpha^*, \alpha^*) - (\Delta + C_p) = \underbrace{(\bar{\pi}_1(\alpha^*) - \Delta)}_{>0} + \underbrace{(\bar{\pi}_2(\alpha^*, \alpha^*) - C_p)}_{<0} = 0. \quad (22)$$

In this case as well, α^* decreases with the cost of the fast trading technology, Δ , because $\Pi^F(\alpha, \alpha)$ decreases in α . Thus, when Δ is larger, α^* must be smaller for (??) to hold. Furthermore, as $\bar{\pi}_2(\alpha^*, \alpha^*) < C_p$ (eq. (??)), we must have $\bar{\pi}_1(\alpha^*) > \Delta$. That is, in equilibrium, the marginal benefit of being fast remains strictly larger than the cost of being fast in equilibrium ($\bar{\pi}_1(\alpha^*)$) when $\Delta < \bar{\Delta}$. Yet speculators just make zero expected profit once the cost of information is taken into account. This emphasizes the importance of accounting for both technological costs and information costs in the evaluation of the profitability of fast traders. Moreover, as $\bar{\pi}_1(\alpha^*) > \Delta$, it must be the case that $\alpha^* < Q$.

Hence, for $\Delta < \bar{\Delta}$, the total mass of speculators decreases with Δ because $\beta^* = \alpha^*$ and α^* decreases with Δ . In contrast, for $\Delta > \bar{\Delta}$, the total mass of speculators increases with Δ because in this case $\beta^* > \alpha^*$ and β^* gets larger when Δ is higher. Thus, the demand for information, β^* , is a U-shape function of the cost of trading fast, Δ . We summarize the results obtained so far in the next proposition. A closed form characterization of $\bar{\Delta}(\theta, C_p)$, α^* , and β^* is given in the proof of the proposition.

Proposition 3. *If $0 < C_p < \frac{\theta}{2}$:*

1. *The demand for information in equilibrium (β^*) is a U-shape function of the cost of trading fast on the signal, Δ and it reaches a minimum for $\Delta = \bar{\Delta}$.*
2. *The demand for the fast trading technology (α^*) decreases with the cost of fast trading, Δ .*
3. *For $\Delta > \bar{\Delta}$, (i) $\alpha^*(\Delta) = \max \{Q(1 - 2\frac{\Delta}{\theta}), 0\}$, (ii) β^* solves (??) and (iii) some speculators never trade on the signal without first processing it ($\beta^* > \alpha^*$).*
4. *For $\Delta \leq \bar{\Delta}$, (i) α^* solves (??), (ii) $\beta^* = \alpha^*$ and (iii) all speculators choose to process information before trading on it ($\beta^* = \alpha^*$).*
5. *There exists a value $C_p^* \in [\frac{\theta}{2}(\frac{1-\theta}{2-\theta}), \frac{\theta}{2}]$ such that if $C_p > C_p^*$ then β^* is maximal for $\Delta = 0$ while if $C_p < C_p^*$ then β^* is maximal when $\Delta \geq \frac{\theta}{2}$ (i.e., when no speculator chooses to trade without processing information).*

Fast trading has two opposite effects on the value of information. On the one hand, it enables speculators to trade multiple times (twice in our model) on their information.

This effect enhances the value of information and therefore the demand for it. On the other hand, by trading fast on information, speculators reduce the expected profit that speculators can obtain after processing information ($\bar{\pi}_2(\alpha, \beta)$ decreases with α ; see Corollary ??). This effect lowers the value of information. It dominates when the cost of the trading technology is larger than $\bar{\Delta}$ while the former effect dominates otherwise. This explains why ultimately the demand for information in equilibrium is a U-shape function of the cost of trading fast.

Figure ?? illustrates this property. It shows that the demand for information as a function of the cost of trading fast for specific parameter values. When $\Delta > \bar{\Delta}$, fast trading technologies and information technologies are substitutes. Indeed, in this case, a decrease in Δ increases the demand for fast trading technologies and, for this reason, lowers the total demand for information (as $\beta^*(\alpha^*)$ decreases when α^* increases for $\Delta > \bar{\Delta}$). In contrast, for $\Delta < \bar{\Delta}$, fast trading technologies and information technologies are complements. Indeed, a reduction in the cost of trading fast (Δ) simultaneously raises the demand for fast trading technologies and information.

Figure ?? also shows that even when the cost of trading fast is zero, the total demand for information can be smaller than when the cost of trading fast is so high that no speculator chooses to trade on information without processing it first, as implied by the last part of Proposition ??.

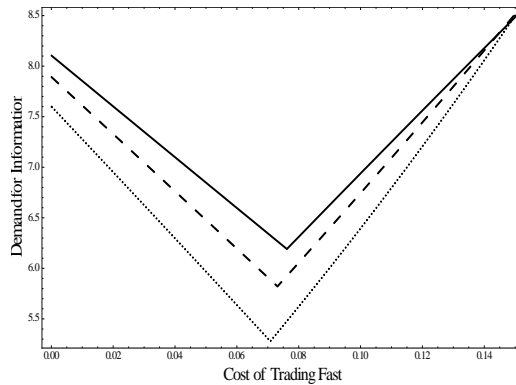


Figure 4: Equilibrium demand for Information and Cost of Trading Fast.
Parameters: $\theta = 0.3$ (dotted line), $\theta = 0.35$ (dashed line), $\theta = 0.4$ (thick line);
 $C_p = 0.1$, $Q = 10$.

Corollary 2. *In equilibrium, other things equal, the mass of fast speculators increases with signal reliability when $\Delta > \bar{\Delta}$. Thus, in this case, the expected size of the immediate price reaction following the signal ($E(|p_1 - \frac{1}{2}|)$) increases with signal reliability. When*

$\Delta < \bar{\Delta}$, the mass of fast speculators may not be a monotonic function of the signal reliability θ (because of corollary ??). However the immediate price reaction following the signal still increases with signal reliability.

This corollary implies that one should see larger price reactions following the arrival of more reliable signals. For instance, news analytics providers assigns a relevance score to each news released to buyers of their service. If news relevance is a proxy for news reliability then the previous corollary implies that more relevant news should move prices more. This is consistent with the findings in von Beschwitz et al.(2013)

In Proposition ??, we focus on the case in which $C_p < \frac{\theta}{2}$. This condition guarantees that, in the absence of fast trading ($\Delta > \frac{\theta}{2}$), some speculators buy information. For completeness, we now consider the case in which $C_p \geq \frac{\theta}{2}$.

Proposition 4. *If $\frac{\theta}{2} \leq C_p < \theta$:*

1. *When $\Delta + C_p > \theta$, there is no demand for information, $\alpha^* = \beta^* = 0$.*
2. *When $\Delta + C_p < \theta$, all speculators choose to be fast in equilibrium $\beta^* = \alpha^*$ and α^* solves equation (??) (see the proof for a closed-form solution).*
3. *The demand for information in equilibrium (β^*) is (weakly) decreasing in the cost of trading fast on information, Δ .*

As $C_p \geq \frac{\theta}{2}$, buying information only to trade once on it is never profitable. Thus, if the cost of trading fast is too large, then no speculator buys information (2nd part of Proposition ??). When the cost of trading fast becomes small enough then buying information becomes attractive as speculators can now trade multiple times on the same signal, which enables them to better amortize the cost of information (3rd part of the proposition). In this case, information acquisition and fast trading are always complements and for this reason the demand for information always declines with the cost of fast trading.

5 Implications

5.1 Price and Trade Patterns

In this section, we analyze in more detail the return and trade patterns induced by speculators' equilibrium behavior. To focus on the interesting case, we assume throughout

that $\Delta < \frac{\theta}{2}$ so that at least some speculators choose to be fast ($\alpha^* > 0$). Otherwise, there is no trading at date $t = 1$. As mentioned previously, fast speculators keep building up their position at date 2 when after processing the signal they learn that it is informative, whether the first period price moved or not. However, if they learn that the signal is false, speculators revert their trades at date 2 only if the first period price moved. Conditional on a false signal, the likelihood of an erroneous price movement is larger when the number of fast speculators is higher, that is, when the cost of trading fast is lower. For this reason, as the next proposition shows, the covariance between speculators' trades at dates 1 and 2 declines when the cost of trading fast declines and can even become negative when this cost is low enough.

Corollary 3. *In equilibrium, the covariance between the trades of speculators at dates 1 and 2 is:*

$$\text{Cov}(x_1, x_2) = \theta - (1 - \theta) \frac{\alpha^*}{Q},$$

This covariance declines when the cost of trading fast becomes smaller because this reduction increases the mass of fast speculators, α^ . It can be positive or negative. For instance, when $\bar{\Delta} < \Delta$, $\text{Cov}(x_1, x_2) < 0$ if and only if $\Delta < \frac{3}{2} - \sqrt{2}$ and $\theta \in [\theta_1(\Delta), \theta_2(\Delta)]$ (where the thresholds $\theta_1(\Delta)$ and $\theta_2(\Delta)$ are defined in the proof of the proposition).*

Thus, a decrease in the cost of trading fast always reduces the covariance between speculators' trades at dates 1 and 2. This covariance can even become negative so that speculators are negatively autocorrelated (both at the speculator-level and at the group level since all speculators trade in the same way at date 2). In our model, trade reversals happen when (i) speculators realize that the signal was false and (ii) the first period price deviates from the expected payoff of the asset conditional on the signal being false. The first condition holds more frequently when θ is low while the second holds more frequently when θ is large. For this reason, as shown by the second part of Corollary ??, the sign of $\text{Cov}(x_1, x_2)$ is non monotonic in θ and is negative for intermediate values of θ and positive for extreme values.

Hirshleifer, Titman and Subrahmanyam (1994) and Brunnermeier (2005) also consider two-periods models with early and late informed investors in which early investors might unwind, at least partially, their position in the second period. However, the source of trade reversals in these models are very different from that in our model. In Hirshleifer, Titman and Subrahmanyam (1994), early informed investors partly unwind their position in the second period because of risk aversion (they optimally share risk with dealers and late informed investors at date 2). This effect is not present in our model because

informed investors are risk neutral. In Brunnermeier (2005), trades reversals are due to heterogeneity speculators' signals. Again this effect is not present in our model because all speculators have the same information. Furthermore, none of these papers predict that the autocorrelation in early and late informed investors should depend on (i) the cost of trading early and (b) the reliability of speculators' signal.

Corollary 4. *In equilibrium, the covariance between the first period return $(p_1 - p_0)$ and the trade of a speculator at date 2 is:*

$$Cov(p_1, x_2) = \theta(2\theta - 1) \frac{\alpha^*}{2Q}.$$

Hence, speculators' orders at date 2 are negatively correlated with the price movement at date 1 iff $\theta < \frac{1}{2}$. Furthermore, a decline in the cost of trading fast, Δ raises the absolute value of the covariance between speculators' trade at date 2 and the first period return because the mass of fast speculators, α^ , is higher when Δ is smaller.*

Conditional on a price movement at date 1, the likelihood that speculators correct this movement at date 2 increases when the likelihood that the signal is false increases (i.e., θ decreases). This explains why for θ small enough, $Cov(p_1, x_2) < 0$. Thus, fast speculators will appear to behave as momentum traders when $\theta > \frac{1}{2}$ and contrarian traders if $\theta < \frac{1}{2}$. Moreover, holding θ constant, Corollary ?? implies that the relationship between past returns and speculators' trades should become stronger when the cost of fast trading declines.

Figures ?? and ?? illustrate the results obtained in this section. They show $Cov(x_1, x_2)$ and $Cov(p_1, x_2)$ as a function of θ for various values of Δ . In each case, covariances are U-shape functions of θ . Furthermore, the covariance in speculators' trades become smaller and even negative as the cost of trading fast declines (see Figure ??). Last, the covariance between speculators' trades at date 2 and the price change at date 1 becomes smaller in absolute value when the cost of trading fast declines (see Figure ??). In Figures ?? and ??, $\Delta < \bar{\Delta}$. This shows that the condition $\Delta > \bar{\Delta}$ is not necessary for the predictions obtained in Corollaries ??.

5.2 Informational Efficiency

As is common in the literature, we measure market efficiency at date t by the average pricing error at this date, that is, $\mathcal{E}_t(\Delta, C_p) = E[(\tilde{V} - P_t)^2]$. Remember that the stock price satisfies, $p_t = E[V|\Omega_t]$, at each date (where Ω_t is the market maker's information

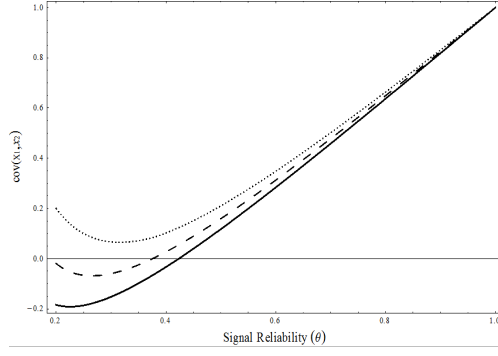


Figure 5: Covariance between speculators' trades at date 1 and 2 as a function of θ . Parameters: $\Delta = 0.1$ (dotted line), $\Delta = 0.05$ (dashed line), $\Delta = 0.01$ (thick line); $C_p = 0.1$, $Q = 10$.

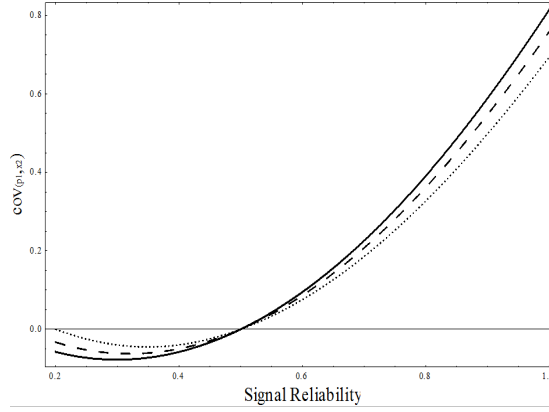


Figure 6: Covariance between the return at date 1 and speculators' trades at date 2 as a function of θ . (Parameters: $\Delta = 0.1$ (dotted line), $\Delta = 0.05$ (dashed line), $\Delta = 0.01$ (thick line); $C_p = 0.1$; $Q = 10$).

set at date t). Thus:

$$\begin{aligned}
 \mathcal{E}_t(\Delta, C_p) &= E[(\tilde{V} - p_t)^2] = E[E[(V - p_t)^2 | \Omega_t]] \\
 &= E[E[(V - E[V | \Omega_t])^2 | \Omega_t]] \\
 &= E[\text{Var}[V | \Omega_t]] \\
 &= E[p_t(1 - p_t)]
 \end{aligned}$$

Hence, the market is more efficient at a given date when the expected conditional variance of the asset payoff at this date ($E[\text{Var}[V | \Omega_t]]$) is lower. Calculations (see the

proof of Corollary ??) show that for $\Delta < \frac{\theta}{2}$ (i.e., $\alpha^* > 0$):

$$\mathcal{E}_1(\Delta, C_p) = \frac{1}{4} - \frac{\theta}{2} \left(\frac{\theta}{2} - \bar{\pi}_1(\alpha^*) \right), \quad (23)$$

and when $\Delta > \frac{\theta}{2}$ (i.e., $\alpha^* = 0$), $\mathcal{V}_1(\Delta) = \frac{1}{4}$. Moreover:

$$\mathcal{E}_2(\Delta, C_p) = \frac{1}{4} - \frac{1}{2} \left(\frac{\theta}{2} - \bar{\pi}_2(\alpha^*, \beta^*) \right). \quad (24)$$

As $\bar{\pi}_2(\alpha^*, \beta^*) \leq C_p < \frac{\theta}{2}$, we always have $0 < \mathcal{E}_2(\Delta, C_p) \leq \frac{1}{4}$. At each date, informational efficiency is higher when equilibrium expected profits for speculators at this date are lower. This is intuitive. Speculators exploit deviations of prices from fundamentals given their information. Thus, their expected profits are larger when prices are less efficient.

Now consider the effect of Δ on informational efficiency. If $\bar{\Delta} < \Delta < \frac{\theta}{2}$, in equilibrium, $\bar{\pi}_1(\alpha^*) = \Delta$ and $\bar{\pi}_2(\alpha^*, \beta^*) = C_p$. Hence, in this case, a small decrease in the cost of fast trading, Δ , leaves unchanged market efficiency at date 2 and increases market efficiency at date 1. If $0 < \Delta < \bar{\Delta}$, a decrease in Δ raises the demand for fast trading technologies and information because (i) α^* is higher when Δ is smaller and (ii) in this case, $\beta^* = \alpha^*$. As, at each date, speculators' expected profit decreases with the mass of speculators at dates 1 and 2, we deduce that in this case a reduction in the cost of fast trading improves informational efficiency both at dates 1 and 2. Hence, we have the following result.

Corollary 5. *A reduction in the cost of fast trading technologies always improve informational efficiency at date 1. Furthermore, it improves informational efficiency at date 2 if $0 < \Delta < \bar{\Delta}$. Otherwise, it has no effect on informational efficiency at date 2 when $\bar{\Delta} \geq \Delta$.*

Critics of high frequency trading usually agree that traders exploiting information at the high frequency enable prices to adjust faster to new information, that is, agree that \mathcal{V}_1 might improve with fast trading. They argue however that it should leave informational efficiency at lower frequency unchanged. The model vindicates this argument when $\Delta > \bar{\Delta}$ but not when $\Delta < \bar{\Delta}$. Indeed, in the former case, a reduction in the cost of fast trading enhances informational efficiency in the very short term but this leaves unchanged informational efficiency after information is processed. In contrast, when $\Delta < \bar{\Delta}$, a reduction in the cost of fast trading improves informational efficiency both in the very short run (before information is processed) and in the longer run, i.e., *after* information is processed.

There are two reasons for this finding. The first, obvious, reason is that when Δ is low enough relative to $\bar{\Delta}$ and $C_p > C_p^*$, the total demand for information, β^* is higher than with no fast trading (see the last part of Proposition ??). However, when $C_p < C_p^*$, the total demand for information is always higher when there is no fast trading at all ($\beta^*(0) > \beta^*(\Delta) \quad \forall \Delta < \frac{\theta}{2}$). Yet, even in this case, it is still the case that informational efficiency at date 2 is higher when $0 < \Delta < \bar{\Delta}$ because there is another mechanism for this result.

This second mechanism is more subtle than the first: fast trading enables the dealer to gradually discover the various dimensions of speculators' informational advantage (the direction of their signal, S , and its actual informativeness, U). Actually, if there is no informed trading at date 1, the market-maker at date 2 faces uncertainty on both the direction of speculators' signal and its actual informativeness, that is, uncertainty on both S and U . In contrast, if some speculators choose to trade fast on information, the market-maker obtains information on the direction of speculators signal in the first period from the order flow in this period. In fact, with probability $\frac{\alpha}{Q}$, the market maker learns S_1 . When this happens, only U remains uncertain in the second period. This facilitates price discovery in the second period, especially if the increase in the mass of fast speculators due to a lower Δ is large enough, that is when $\Delta < \bar{\Delta}$. This second mechanism always plays a role as long as $\theta < 1$, that is, there is uncertainty on U .

This beneficial effect of fast trading on informational efficiency will persist in a model with more trading rounds as long as it takes time for speculators to fully process a signal. For instance, suppose that there are N trading rounds before the uncertainty on the asset payoff is resolved. At date $n = 1$, speculators receive signal S_1 :

$$S_1 = U_1 S_2 + (1 - U_1) \epsilon_1.$$

At date 2, before trading, speculators learn whether $U_1 = 1$ or $U_1 = 0$. In the latter case, they know that the signal is false with certainty. If instead $U_1 = 1$, they must keep processing the signal (make further investigation) to decide whether the signal is false or not. Specifically, they observe $S_2 = U_2 S_3 + (1 - U_2) \epsilon_2$ where U_2 is distributed as U_1 . This process is repeated until date $N - 1$. At the first date, $n + 1$, at which $U_n = 0$, speculators discover that the initial signal was false. If there is no such date, they know with certainty the payoff of the asset at date $t = N$ (set $S_N = V$). Therefore it can take up to N trading rounds for investors to fully process the signal received just before trading at date 1. Then, in this case, early trading by informed investors (even on very

noisy signal) will enable the dealer to better learn about the realizations of S_1, U_1, U_2 etc. and thereby will make the market eventually more efficient. For signals that are difficult to process, N can be large. In this case, fast trading on information might eventually contribute to market efficiency in the “long run.”

5.3 Price Reversals and Mini-Flash Crashes

The analysis of the previous section shows that a reduction in the cost of fast trading (and thereby an increase in the number of speculators who trade before processing signals) improves informational efficiency at date 1, and can even improve it at date 2. These findings are consistent with recent empirical findings about the effects of high frequency traders on price discovery (in particular Brogaard, Hendershott, and Riordan (2013)). Yet, as mentioned in the introduction, market participants have expressed concerns that fast trading makes financial markets less stable. In particular, the rise of high frequency trading seems to coincide with more frequent sharp price movements followed by quick price reversals. At first glance, this possibility seems inconsistent with academic findings regarding the effects of high frequency traders on price discovery. We now show that this is not necessarily the case because a reduction in the cost of trading fast can simultaneously make the market more information efficient (as shown in the previous section) and increase the likelihood of large price reversals (as shown below).

In our model, price changes are serially uncorrelated because prices are semi-strong form efficient at each date ($p_t = E[V|\Omega_t]$). Yet, conditional on the arrival of a false signal ($U = 0$), price reversals occur when some speculators choose to trade fast on information ($\alpha^* > 0$). To see this, consider Figure ?? and suppose that $S_1 = 0$. In this case with probability $\frac{\alpha}{Q}$, there is a strong sell order imbalance at date $t = 1$ and the price falls at $p_1 = \frac{1-\theta}{2}$. Now suppose that the signal turns out to be false ($U = 0$). In this case speculators realize that the current price level (p_1) is not in line with the fundamental value of the asset and buys it back. With probability $\frac{\beta}{Q}$, this buying pressure is strong enough and the price of the asset reverts to its unconditional value $\frac{1}{2}$. With probability $(1 - \frac{\beta}{Q})$, this buying pressure is not sufficient to correct the mispricing and the price of the asset remains at $p_1 = \frac{1-\theta}{2}$. However, on average, the price at date 3 will be $\frac{1}{2}$ and therefore the price will eventually revert at $\frac{1}{2}$ on average. Conditional on $\{S_1 = 1, U = 0\}$, price movements are symmetric.

Thus, conditional on the initial signal being false, the price movement from $t = 0$ to $t = 1$ reverts with probability $\frac{\alpha^*}{Q^2}$ in equilibrium. Furthermore, conditional on a reversal,

this reversal is *quick* (i.e., take place before $t = 3$) with probability $\frac{\alpha^*\beta^*}{2Q^2}$. Hence, the unconditional likelihood of a price reversal is:

$$p_{Reversal}(\Delta, \theta) = (1 - \theta) \frac{\alpha^*}{Q}, \quad (25)$$

and the unconditional likelihood of a quick price reversal is:

$$p_{Quick\ Reversal}(\Delta, \theta) = (1 - \theta) \frac{\alpha^*\beta^*}{Q^2}. \quad (26)$$

Obviously, these probabilities are strictly positive iff $\alpha^* > 0$. Thus, price reversals occur only if some speculators choose to trade before processing information, which requires $\Delta < \frac{\theta}{2}$. Under this condition, we first study the effect of θ and Δ on the likelihood of a reversal at any horizon (short, i.e., $t = 2$ or longer, i.e., $t = 3$).

Corollary 6. *1. Holding θ fixed, the likelihood of a price reversal, $p_{Reversal}(\Delta, \theta)$, is zero when $\Delta > \frac{\theta}{2}$ and positive when $\Delta < \frac{\theta}{2}$. In this case, the likelihood of a price reversal is inversely related to the cost of trading fast, Δ ($\frac{\partial p_{Reversal}(\Delta, \theta)}{\partial \Delta}$ when $\Delta < \frac{\theta}{2}$).*

2. When $\bar{\Delta} < \Delta < \theta/2$, the likelihood of a price reversal, $p_{Reversal}(\Delta, \theta)$, is an inverse U-shape function of signal reliability θ with a maximum for $\theta = \sqrt{2\bar{\Delta}}$.

When fast trading becomes less costly, the mass of fast speculators increases. As a result, trades are more likely to move prices at date 1. Thus, holding fixed the frequency of erroneous signals (θ), the likelihood of price reversals become larger when the cost of fast trading decreases, as claimed in the first part of the proposition.

More surprisingly, as shown by the second part of the proposition, an increase in the reliability of speculators' signals can also generate an increase in the unconditional likelihood of a reversal when $\bar{\Delta} < \Delta < \frac{\theta}{2}$ since $p_{Reversal}(\Delta, \theta)$ then peaks at $\theta = \sqrt{2\bar{\Delta}}$ (holding Δ constant). When reliability increases, the signal received by speculators is false less frequently. However, a greater mass of speculators choose to trade at date 1 because waiting to process information before trading has less value. As a result, a larger mass of speculators trades on the signal when it is false, making the likelihood of an erroneous price movement at date 1 larger. This effect dominates the former when $\theta < \sqrt{2\bar{\Delta}}$.

Thus, a reduction in the cost of trading fast causes an increase in the likelihood of price reversals. In addition, the model implies that the horizon over which price reversals take place should be shorter when fast trading becomes less costly, as claimed in the next

proposition.

Corollary 7. *Suppose $\Delta < \bar{\Delta}$. Holding θ fixed, the likelihood of a quick price reversal, $p_{\text{quick Reversal}}(\Delta, \theta)$ increases when the cost of trading fast, Δ , decreases.*

Occurrences of quick price reversals after a false signal requires combining two forces. First, one needs to have a large number of speculators, α^* , reacting to a false signal. Indeed, this increases the chance that the buying or selling pressure following signal arrival will be strong enough to trigger an erroneous price change. Second, one needs to have a large number of speculators, β^* correcting this erroneous price change, once speculators realize that the signal is false. When $\Delta < \bar{\Delta}$, a reduction in the cost of trading fast strengthens these two forces because it increases both α^* and β^* (see Proposition ??).

When $\Delta > \bar{\Delta}$, a reduction in the cost of trading fast increases the number of fast speculators, α^* but it reduces the number of speculators trading after processing information, β^* . Thus, the net effect of a reduction in Δ on the likelihood of a quick price reversal is ambiguous. However, we have checked through numerical simulations that the first effect tends to dominate so that Corollary ?? still holds when $\bar{\Delta} < \Delta < \frac{\theta}{2}$.

Figure ?? illustrates this point. It shows the likelihood of a quick price reversal as a function of θ for various values of the cost of trading fast. For the parameter values in Figure ??, Condition $\Delta < \bar{\Delta}$ is not always satisfied. Yet, it is still the case that the likelihood of a quick price reversal is inversely related to the cost of trading fast Δ . Moreover, this likelihood is an inverse U-shape function of θ , just as the likelihood of a price reversal is.

Corollary ?? suggests a possible explanation for the perception that mini-flash crashes are more frequent. A reduction in the cost of fast trading implies that more speculators choose to trade on information without processing it. As a result, holding θ constant, the likelihood of a price movement followed by a quick correction of this movement gets larger. If there are time-variations in θ for a given stock, the initial price movement can be large when speculators are quite confident that the initial signal is informative (that is when θ is large). This type of situation can generate “mini flash crashes”: a sharp drop (or spike) in prices followed by a very quick correction.

To formalize this idea, consider the following extension of the model in which θ is stochastic. Specifically, at date $t = 0^+$, θ is drawn from a distribution with support $[0, 1]$ and mean $\bar{\theta}$. All market participants observe θ but speculators must decide to acquire

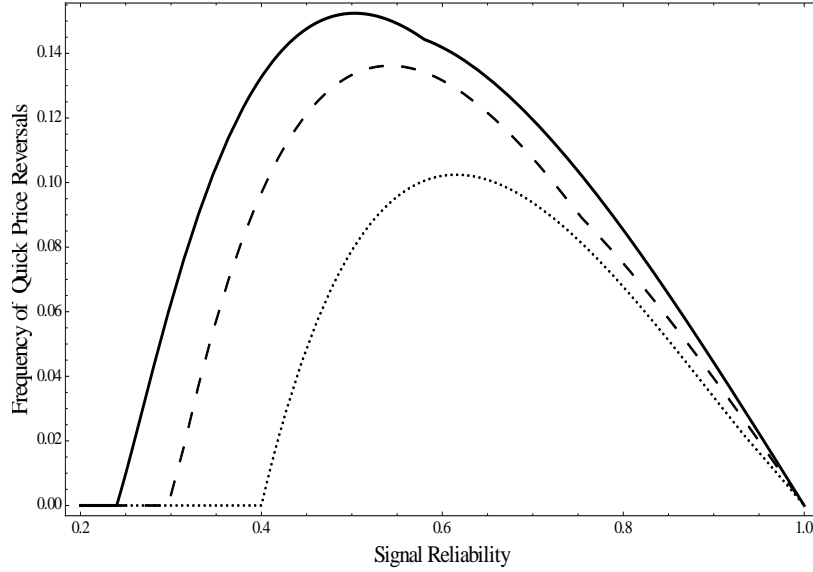


Figure 7: Probability of a quick price reversal as a function of θ . Parameters: $\Delta = 0.2$ (dotted line), $\Delta = 0.15$ (dashed line), $\Delta = 0.12$ (thick line); $C_p = 0.1$, $Q = 10$.

information and to be fast before observing θ .¹⁶ The idea is that there are time-variations in speculators' signal reliability for a given asset and when they make their decision to buy information about an asset speculators only know the distribution of the reliability of the information they will receive.

Empirically, it is natural to define a "mini flash crash" as a price movement that is (i) large relative to some measure of normal return volatility for the asset and (ii) quickly corrected, that is, followed by a similar price movement in the opposite direction. In our model, the standard deviation of the asset return from date 0 to date 3 is $\frac{1}{2}$. Hence, we say that a mini-flash crash happens if the following conditions are satisfied:

1. The change in price from date $t = 1$ to date $t = 2$ has a sign opposite to the change in price from date $t = 0$ to date $t = 1$.
2. The size of this quick reversal is larger than $\frac{R}{2}$ where $0 \ll R \leq 1$.

As speculators must decide to invest in the fast trading technology at date 0, the analysis of the determination of α^* and β^* follows the same steps as in Proposition ??.

¹⁶For tractability, we assume that when a signal arrives the market-maker knows that the signal has arrived and the reliability of this signal, θ . This assumption is implicit in many models of trading with asymmetric information (e.g., Glosten and Milgrom (1985)).

The only difference is that α^* and β^* are determined by expected profits taken over all values of θ , as shown in Appendix B. In particular, β^* remains a U-shape function of Δ with a minimum for $\Delta = \bar{\Delta}^r$. Moreover, for $\bar{\Delta}^r < \Delta < \frac{\bar{\theta}}{2}$, we have:

$$\alpha^* = Q \left(1 - \frac{2}{\bar{\theta}} \Delta \right).$$

and β^* solves:

$$E_{\theta}[\bar{\pi}_2(\alpha^*, \beta^*)] = C_p,$$

while for $\Delta < \bar{\Delta}$, we have $\alpha^* = \beta^*$ such that:

$$E_{\theta}[\Pi^F(\alpha^*, \alpha^*)] = C_p + \Delta,$$

Conditional on the realization of a signal reliability, the likelihood of a mini flash-crash is then given by:

$$p_{crash}^c(\theta) = \frac{\alpha^* \beta^*}{4Q^2} (1 - \theta) \mathbb{I}_{\theta > R},$$

where $\mathbb{I}_{\theta > R}$ is the indicator function. The unconditional probability of a mini-flash crash is therefore:

$$p_{crash} = \int_0^1 p_{crash}^c(x) f(x) dx = \frac{\alpha^* \beta^*}{4Q^2} \int_R^1 (1 - x) f(x) dx,$$

which yields:

$$p_{crash} = \frac{\alpha^* \beta^*}{4Q^2} (1 - E[\theta | \theta > R]) Pr[\theta > R]. \quad (27)$$

First, obviously, the likelihood of a flash crash declines when R gets larger, that is, when one imposes a larger threshold on the size of a price reversal to categorize this reversal as a mini-flash crash. Second, when $\Delta < \bar{\Delta}^r$, the likelihood of a mini flash crash increases when the cost of trading fast decreases because α^* and β^* then increases. This is the mechanism leading to Corollary ?? when θ is constant.

To gain further insight on the determinants of the likelihood of a flash crash in our setting, we use numerical simulations.¹⁷ We assume that $\theta = X^{\lambda}$, where X is a random variable drawn from the uniform distribution on $[0, 1]$ and $\lambda \in [0, +\infty)$ is a real number. In this particular case, θ is distributed on $[0, 1]$. The higher is λ , the more likely are small realizations of θ . To see this, let g and G be the pdf and cdf of θ . We have:

$$G(x) = Pr[X < x^{\frac{1}{\lambda}}] = x^{\frac{1}{\lambda}},$$

¹⁷We explain how we perform these simulations in the last part of Appendix B.

$$g(x) = \frac{1}{\lambda} x^{\frac{1}{\lambda}-1}.$$

Thus, a signal with $\lambda = \lambda_0$ dominates, in the sense of first order stochastic dominance, a signal with λ_1 iff $\lambda_1 > \lambda_0$. That is, increasing λ makes signals with a low reliability more likely. We have:

$$\begin{aligned}\bar{\theta} &= E[X^\lambda] = \int_0^1 x^\lambda dx = \frac{1}{\lambda+1}, \\ Pr[\theta > R] &= 1 - R^{\frac{1}{\lambda}}, \\ \bar{\theta} = E[\theta|\theta > R] &= \frac{\int_{R^{\frac{1}{\lambda}}}^1 x^\lambda dx}{1 - R^{\frac{1}{\lambda}}} = \frac{1}{\lambda+1} \frac{1 - R^{\frac{1}{\lambda}+1}}{1 - R^{\frac{1}{\lambda}}}.\end{aligned}$$

When λ increases, $\bar{\theta}$ and $E[\theta|\theta > R]$ decrease because large realizations of θ are less likely. Thus, λ^{-1} plays the role of θ in the baseline model. As there is a one-to-one mapping between λ and $\bar{\theta}$, we can express λ as a function of $\bar{\theta}$ and use the expected reliability rather than λ as the choice parameter for the distribution of θ (keeping in mind that increasing $\bar{\theta}$ changes the entire distribution of θ). Using this observation, we obtain that the likelihood of a mini flash crash with this parametrization is:

$$p_{crash} = \frac{\alpha^* \beta^*}{4Q^2} [1 - \bar{\theta} - R^{\frac{\bar{\theta}}{1-\bar{\theta}}} (1 - \bar{\theta}R)].$$

Based on Corollaries ?? and ??, we expect the likelihood of a flash crash to increase when the cost of trading fast decreases (Δ smaller) and to be an inverse U-shape function of $\bar{\theta}$. Numerical simulations show that this intuition is correct.

As an example, consider Figure ?. It shows that the likelihood of a mini-flash crash is an inverse U-shape function of the mean reliability of speculators' signals. Thus, mini-flash crashes are more likely for stocks for which news are neither too unreliable, nor too reliable. Moreover, Figure ? shows that the likelihood of a mini-flash crash increases, other things equal, when the cost of trading fast on information gets smaller. Interestingly, the likelihood of a mini-flash crash *per news* can be quite large even for conservative values of R . For instance, for $R = 70\%$ and $\Delta = 0.1$, the likelihood of a mini flash-crash can be as high as 10% and it peaks for stocks in which news tend to be very accurate on average ($\bar{\theta} \approx 0.9$).

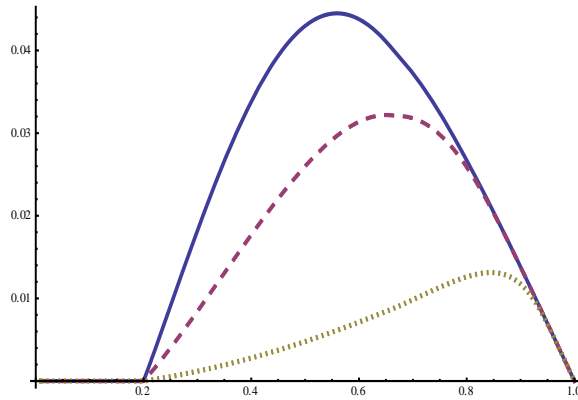


Figure 8: Likelihood of a quick price reversal as a function of the mean signal reliability $\bar{\theta}$ for different values of R : $R = 10\%$ (plain line), $R = 30\%$ (dashed line), and $R = 70\%$ (dotted line). In each case, $\Delta = 0.1$, $C_p = 0.06$, and $Q = 10$.

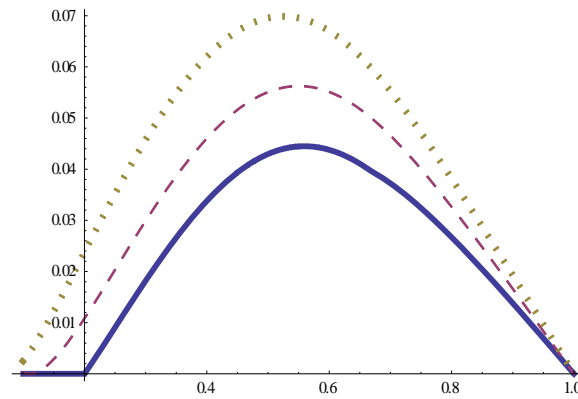


Figure 9: Likelihood of a quick price reversal as a function of the mean signal reliability $\bar{\theta}$ for different values of Δ : $\Delta = 0.1$ (plain line), $\Delta = 0.05$ (dashed line), and $\Delta = 0.01$ (dotted line). In each case, $R = 10\%$, $C_p = 0.06$, and $Q = 10$.

6 Conclusion

We have considered a model in which speculators can discover whether a signal is true or false by processing it but this takes time. Hence they face a trade-off between trading fast on unreliable signals, at the risk of trading on false signals, or trading after processing the signal, at the risk of losing an opportunity. The model generates a rich set of testable implications regarding patterns of trades and prices following the arrival of private signals (e.g., the release of news to a subset of investors who buy access to these news). In particular the model implies that a reduction in the cost of trading fast should:

1. Increase the frequency of quick price reversals, including large price reversals (“mini-flash crashes”).
2. Lower the covariance in speculators’ trades observed before and after processing news and even potentially generate a negative autocorrelation in these trades.
3. Increase the absolute covariance in speculators’ trades after processing news and past returns.
4. Reduce or leave unchanged average pricing errors at each trading date, even though a reduction in the cost of trading fast increases the likelihood of trading on false news.

The first and the last implication means that a decline in the cost of trading fast can, paradoxically, simultaneously makes financial markets more efficient but more prone to large and transient price reversals. Thus, the model offers an explanation for two apparently incompatible observations: (i) the perception by market participants that large and quick price reversals (mini-flash crashes) are more frequent and (ii) academic findings suggesting that high frequency traders contribute to price discovery.

In our model, prices are set by rational traders taking into account all publicly available information when they set their quotes. Hence, unconditionally, change in prices are not serially correlated. Thus, price and trade patterns predicted by our model are not due to deviations of prices from fair values given available information (as for instance in Daniel, Hirshleifer, and Subrahmanyam (1998)). They are just a consequence of the fact that (i) news are not perfectly reliable (they might be false), (ii) speculators learn progressively about whether the news they receive are false or not and (iii) optimally choose to trade on signals before processing them or not. Overall, the model suggests that empirical analyses relating news to returns should account for news reliability and the possibility that some news on which participants trade are indeed false. These features could explain systematic patterns in returns and trades observed in the data.

Appendix A

Proof of proposition ??.

Stock Price. We first derive the equilibrium stock price when speculators behave as described in part 1 of the proposition. As explained in the text:

$$p_1(f_1) = Pr[V = 1 | \tilde{f}_1 = f_1] = \frac{Pr[\tilde{f}_1 = f_1 | V = 1]Pr[V = 1]}{Pr[\tilde{f}_1 = f_1]}. \quad (28)$$

Fast speculators buy the asset at date 1 when they observe $S_1 = 1$. Hence, conditional on $V = 1$, aggregate speculators' demand is α with probability $(1 + \theta)/2$ and $-\alpha$ with probability $(1 - \theta)/2$. Thus:

$$Pr[\tilde{f}_1 = f_1 | V = 1] = \left(\frac{1 + \theta}{2}\right)\phi(f_1 - \alpha) + \frac{1 - \theta}{2}\phi(f_1 + \alpha). \quad (29)$$

Furthermore, by symmetry:

$$Pr[\tilde{f}_1 = f_1] = \frac{1}{2}\phi(f_1 - \alpha) + \frac{1}{2}\phi(f_1 + \alpha). \quad (30)$$

Substituting (??) and (??) in (??) and using the fact that $Pr[V = 1] = 1/2$, we obtain (??).

Trading strategies. Consider a fast speculator first. For a given trade x_1 , his expected profit at date 1 when he observes signal $S_1 = s$ is:

$$\pi_1(\alpha, s) = x_1(\mu(S_1) - E[p_1 | S_1 = s]).$$

Now remember that $p_1 = E[V | \tilde{f}_1]$. As the market-maker's information set at date 1 is coarser than speculators' information set, we have:

$$\mu(0) \leq p_1 \leq \mu(1),$$

with a strict inequality when $f_1 \in [-Q + \alpha, Q - \alpha]$ because in this case the order flow at date 1 contains no information. We deduce that:

$$\mu(0) < E[p_1 | S_1] < \mu(1),$$

when $\alpha \leq Q$. Thus, in this case, it is a strictly dominant strategy for a speculator to buy the asset when $S_1 = 1$ and sell the asset when $S_1 = 0$. It follows that the equilibrium at date 1 is unique when $\alpha \leq Q$.

When $\alpha \geq Q$, $[-Q + \alpha, Q - \alpha]$ is an empty set. Thus, the order flow is fully revealing and $p_1 = \mu(S_1)$. Hence, a speculator obtains a zero expected profit for all x_1 whether $S_1 = 1$ or $S_1 = 0$. Buying the asset when $S_1 = 1$ and selling the asset when $S_1 = 0$ is then weakly dominant.

Speculators' Expected Profit in Period 1. If a speculator receives the signal $S_1 = 1$ and $\alpha \leq Q$, his expected profit is:

$$\begin{aligned}
\pi_1(\alpha, 1) &= \int_{[-Q, Q]} \left[\frac{1 + \theta}{2} - \frac{(1 + \theta)\phi(l_1) + (1 - \theta)\phi(l_1 + 2\alpha)}{\phi(l) + \phi(l + 2\alpha)} \times \frac{1}{2} \right] \phi(l_1) dl_1, \\
&= \int_{[-Q, Q]} \frac{\theta\phi(l_1 + 2\alpha)}{\phi(l_1) + \phi(l + 2\alpha)} \phi(l_1) dl_1, \\
&= \int_{[-Q + \alpha, Q + \alpha]} \frac{\theta\phi(l_1 + \alpha)}{\phi(l - \alpha) + \phi(l + \alpha)} \phi(l_1 - \alpha) dl_1, \\
&= \int_{[-Q + \alpha, Q - \alpha]} \frac{\theta\phi(l_1 + \alpha)}{\phi(l - \alpha) + \phi(l + \alpha)} \phi(l_1 - \alpha) dl_1, \\
&= \int_{[-Q + \alpha, Q - \alpha]} \frac{\theta}{2} dl_1, \\
&= \frac{\theta}{2} \left(\frac{Q - \alpha}{Q} \right).
\end{aligned}$$

because $\phi(l_1 + \alpha) = 0$ for $l_1 > Q - \alpha$. By symmetry, this is also a speculator's expected profit when he receives a signal $S_1 = 0$. As $S_1 = 1$ is as likely as $S_1 = 0$, we deduce that, for $\alpha \leq Q$,

$$\bar{\pi}_1(\alpha) = \frac{\theta}{2} \left(\frac{Q - \alpha}{Q} \right).$$

For $\alpha > Q$, the order flow at date 1 fully reveals speculators' signal and accordingly speculators' expected profit is zero. Hence, speculators' expected profit is:

$$\bar{\pi}_1(\alpha) = \frac{\theta}{2} \left(\text{Max} \left\{ \frac{Q - \alpha}{Q}, 0 \right\} \right).$$

Proof of Proposition ??.

Step 1. Stock Price. We first derive the equilibrium stock price when speculators behave as described in part 1 of Proposition ???. As explained in the text:

$$p_2(f_2, f_1) = Pr[V = 1 | \tilde{f}_2 = f_2, \tilde{f}_1 = f_1] = \frac{Pr[\tilde{f}_2 = f_2, \tilde{f}_1 = f_1 | V = 1]}{Pr[\tilde{f}_2 = f_2, \tilde{f}_1 = f_1]}. \quad (31)$$

Conditional on $V = 1$, three events can happen at date 2. If $U = 1$, speculators learn that the signal was correct in period 1. As $V = 1$, this means that speculators observed $S_1 = 1$ at date 1. In this case, their aggregate demand at dates 1 and 2 are α and β , respectively. If $U = 0$, speculators learn that their signal was in fact incorrect. Thus, speculators' aggregate demand in period 2 is M_0 . With probability $1/2$, speculators observed $S_1 = 1$ in period 1 and with probability $1/2$, they observed $S_1 = 0$. In the first case, speculators' aggregate demand for the asset is α and in the second case, speculators' aggregate demand is $-\alpha$. We deduce that:

$$\begin{aligned} Pr[\tilde{f}_2 = f_2, \tilde{f}_1 = f_1 | V = 1] = \\ \phi(f_2 - \beta)\phi(f_1 - \alpha)\theta + \frac{1}{2}(\phi(f_2 - M_0)\phi(f_1 - \alpha)(1 - \theta) + \phi(f_2 - M_0)\phi(f_1 + \alpha)(1 - \theta)). \end{aligned} \quad (32)$$

Furthermore, by symmetry:

$$\begin{aligned} Pr[\tilde{f}_2 = f_2, \tilde{f}_1 = f_1] = \\ \frac{1}{2}(\phi(f_2 - \beta)\phi(f_1 - \alpha)\theta + \phi(f_2 + \beta)\phi(f_1 + \alpha)\theta + \phi(f_2 - M_0)\phi(f_1 - \alpha)(1 - \theta) + \phi(f_2 - M_0)\phi(f_1 + \alpha)(1 - \theta)). \end{aligned} \quad (33)$$

We deduce that:

$$p_2(f_2, f_1) = \frac{\theta\phi(f_1 - \alpha)\phi(f_2 - \beta) + \frac{1-\theta}{2}[\phi(f_1 - \alpha) + \phi(f_1 + \alpha)]\phi(f_2 - M_0(f_1))}{\theta[\phi(f_1 - \alpha)\phi(f_2 - \beta) + \phi(f_1 + \alpha)\phi(f_2 + \beta)] + (1 - \theta)[\phi(f_1 - \alpha) + \phi(f_1 + \alpha)]\phi(f_2 - M_0(f_1))} \quad (34)$$

If $\beta \leq Q$, this means:

$$\text{Case A1: if } f_1 \in [-Q - \alpha, -Q + \alpha), p_2(f_2, f_1) = \begin{cases} 0 & \text{if } f_2 \in [-Q - \beta, -Q + \beta) \\ \frac{1-\theta}{2} & \text{if } f_2 \in [-Q + \beta, Q - \beta] \\ \frac{1}{2} & \text{if } f_2 \in (Q - \beta, Q + \beta] \end{cases}$$

$$\text{Case B1: if } f_1 \in [-Q + \alpha, Q - \alpha], p_2(f_2, f_1) = \begin{cases} 0 & \text{if } f_2 \in [-Q - \beta, -Q) \\ \frac{1-\theta}{2-\theta} & \text{if } f_2 \in [-Q, -Q + \beta) \\ \frac{1}{2} & \text{if } f_2 \in [-Q + \beta, Q - \beta] \\ \frac{1}{2-\theta} & \text{if } f_2 \in (Q - \beta, Q] \\ 1 & \text{if } f_2 \in (Q, Q + \beta] \end{cases}$$

$$\text{Case C1: if } f_1 \in (Q - \alpha, Q + \alpha], p_2(f_2, f_1) = \begin{cases} \frac{1}{2} & \text{if } f_2 \in [-Q - \beta, -Q + \beta) \\ \frac{1+\theta}{2} & \text{if } f_2 \in [-Q + \beta, Q - \beta] \\ 1 & \text{if } f_2 \in (Q - \beta, Q + \beta] \end{cases}$$

If instead $Q < \beta \leq 2Q$, we have:

$$\text{Case A2: if } f_1 \in [-Q - \alpha, -Q + \alpha), \quad p_2(f_2, f_1) = \begin{cases} 0 & \text{if } f_2 \in [-Q - \beta, Q - \beta] \\ \frac{1}{2} & \text{if } f_2 \in [-Q + \beta, Q + \beta] \end{cases}$$

$$\text{Case B2: if } f_1 \in [-Q + \alpha, Q - \alpha], \quad p_2(f_2, f_1) = \begin{cases} 0 & \text{if } f_2 \in [-Q - \beta, -Q) \\ \frac{1-\theta}{2-\theta} & \text{if } f_2 \in [-Q, Q - \beta) \\ \frac{1}{2} & \text{if } f_2 \in [Q - \beta, -Q + \beta] \\ \frac{1}{2-\theta} & \text{if } f_2 \in (-Q + \beta, Q] \\ 1 & \text{if } f_2 \in (Q, Q + \beta] \end{cases}$$

$$\text{Case C2: if } f_1 \in (Q - \alpha, Q + \alpha], \quad p_2(f_2, f_1) = \begin{cases} \frac{1}{2} & \text{if } f_2 \in [-Q - \beta, Q - \beta] \\ 1 & \text{if } f_2 \in (-Q + \beta, Q - \beta] \end{cases}$$

Step 2. Speculators' Trading strategies. If $U = 1$, speculators' trading strategies are as at $t = 1$. As the proof is similar to that at $t = 1$, we skip it for brevity and only discuss the case $U = 0$ now. We show that a speculator's optimal trading strategy is as described in the first part of Proposition ?? when (a) he expects other speculators to behave as described in the first part of Proposition ?? and (b) the stock price to be given by (??).

The expected profit of a speculator who trades x_2 shares at date 2 when $U = 0$ and $S_1 = s$ is:

$$\pi_2(\alpha, \beta, s, 0) = x_2(E[V|U = 0, S_1 = s] - E[p_2|U = 0, S_1 = s, p_1]) = x_2 \times \Sigma(f_1, M_0), \quad (35)$$

where

$$\Sigma(f_1, M_0) = \int_{[-Q-\beta, Q+\beta]} \left[\frac{1}{2} - p_2(f_2, f_1) \right] \phi(f_2 - M_0) df_2,$$

is the expected difference between the speculator's valuation of the asset conditional on $U = 0$ (i.e., $1/2$) and the speculator's expectation of the stock price given other speculators' aggregate demand, M_0 .

Suppose first that $\beta \leq Q$. Using (??), we deduce that:

$$\Sigma(f_1, M_0) = -\frac{1}{2} \int \frac{\theta[\phi(f_1 - \alpha)\phi(f_2 - \beta) - \phi(f_1 + \alpha)\phi(f_2 + \beta)]\phi(f_2 - M_0)}{\theta[\phi(f_1 - \alpha)\phi(f_2 - \beta) + \phi(f_1 + \alpha)\phi(f_2 + \beta)] + (1 - \theta)[\phi(f_1 - \alpha) + \phi(f_1 + \alpha)]\phi(f_2 - M_0)} df_2$$

which is equal to

$$\Sigma(f_1, M_0) = -\frac{1}{2Q} \int_{[-Q-\beta, Q+\beta]} \frac{1}{2} \frac{N(f_1, f_2)}{D(f_1, f_2)} df_2,$$

with

$$N(f_1, f_2) = \theta[\mathbb{I}_{\{f_1 \in [-Q+\alpha, Q+\alpha]\}} \mathbb{I}_{\{f_2 \in [-Q+\beta, Q+\beta]\}} - \mathbb{I}_{\{f_1 \in [-Q-\alpha, Q-\alpha]\}} \mathbb{I}_{\{f_2 \in [-Q-\beta, Q-\beta]\}}] \mathbb{I}_{\{f_2 \in [-Q+M_0, Q+M_0]\}},$$

and

$$D(f_1, f_2) = \theta[\mathbb{I}_{\{f_1 \in [-Q+\alpha, Q+\alpha]\}} \mathbb{I}_{\{f_2 \in [-Q+\beta, Q+\beta]\}} + \mathbb{I}_{\{f_1 \in [-Q-\alpha, Q-\alpha]\}} \mathbb{I}_{\{f_2 \in [-Q-\beta, Q-\beta]\}}] + (1 - \theta)[\mathbb{I}_{\{f_1 \in [-Q+\alpha, Q+\alpha]\}} + \mathbb{I}_{\{f_1 \in [-Q-\alpha, Q-\alpha]\}}] \mathbb{I}_{\{f_2 \in [-Q+M_0, Q+M_0]\}}.$$

Now suppose that $f_1 \in [-Q - \alpha, -Q + \alpha)$. In this case, the first period stock price is $p_1 = \mu(0) < 1/2$ and each speculator expects other speculators to buy the asset ($M_0 = \beta$). Thus:

$$\frac{N(f_1, f_2)}{D(f_1, f_2)} = \begin{cases} 0 & \text{if } f_2 \in [-Q - \beta, -Q + \beta), \\ +1 & \text{if } f_2 \in [-Q + M_0, Q - \beta], \\ 0 & \text{if } f_2 \in (Q - \beta, Q + \beta]. \end{cases}$$

Consequently

$$\Sigma(f_1, M_0) = \theta \frac{Q - \beta}{2Q} > 0.$$

Hence, a speculator's expected profit is maximized when $x_2 = 1$, as prescribed by the strategy described in the first part of the proposition.

The case in which $f_1 \in]Q - \alpha, Q + \alpha]$ is symmetric. In this case, the first period stock price is $p_1 = \mu(1)$ and the speculator expects other speculators to sell the asset when $U = 0$. Following the same steps as when $f_1 \in [-Q - \alpha, -Q + \alpha)$, we conclude that the speculator's expected profit is maximized when $x_2 = -1$.

If $f_1 \in [-Q + \alpha, Q - \alpha]$, the first period price is $p_1 = 1/2$ and the speculator expects other

speculators not to trade ($M_0 = 0$). Hence:

$$\frac{N(f_1, f_2)}{D(f_1, f_2)} = \begin{cases} 0 & \text{if } f_2 \in [-Q - \beta, -Q[\\ +\frac{\theta}{2-\theta} & \text{if } f_2 \in [-Q, -Q + \beta[\\ 0 & \text{if } f_2 \in [-Q + \beta, Q - \beta] \\ -\frac{\theta}{2-\theta} & \text{if } f_2 \in]Q - \beta, Q] \\ 0 & \text{if } f_2 \in]Q, Q + \beta] \end{cases}$$

Then

$$\Sigma(f_1, M_0) = \left(\frac{\theta}{2-\theta}\beta\right) - \frac{\theta}{2-\theta}\beta = 0.$$

Hence, the speculator's expected profit is zero for any $x_2 \in \{-1, 0, 1\}$. Thus, $x_2 = 0$ is a best response in this case.

In sum, we have shown that, if $\beta \leq Q$, the trading strategies described in Part 1 of Proposition ?? when $U = 0$ form an equilibrium. Now suppose that $Q \leq \beta \leq 2Q$. If $f_1 \in [-Q - \alpha, -Q + \alpha)$ then the order flow in the first period is fully revealing and the market maker infers that $S_1 = -1$. In this case, if $U = 1$, then $M_0 = \beta$ in the second period and in this case $f_2 \in [-Q - \beta, Q - \beta]$. As $Q - \beta \leq -Q + \beta$, the market maker infers that speculators in the second period observed $U = 1$ and sets $p_2 = 0$ (see Case A2 in Step 1). Hence, the speculator's expected profit is zero. The case $f_1 \in [Q - \alpha, Q + \alpha)$ is symmetric.

If $f_1 \in [-Q + \alpha, Q - \alpha]$ ($p_1 = 1/2$), then the order flow in the first period contains no information. Furthermore, the order flow in the second period is not fully revealing as well (Case B2 in Step1) as long as $\beta \leq 2Q$. In this case, we can follow the same steps as when $\beta \leq Q$ to show that the trading strategy described in the first part of the proposition is a best response for each speculator when each expects other speculators to follow this strategy and the stock price is given by (??).

Step 3. Speculators' Expected Profit. As a good and a bad signal are equally likely, speculators' expected profit at date 2 is identical when $S_1 = 1$ and $S_1 = 0$. Thus, we just need to compute a speculator's expected profit conditional on $S_1 = 1$. When $S_1 = 1$, all fast speculators buy the asset at date 1. Thus, $f_1 \in [-Q + \alpha, Q + \alpha]$. Suppose first that $\beta \leq Q$.

Case 1. If $f_1 \in [-Q + \alpha, Q - \alpha]$, the order flow contains no information at date 1. Speculators

buy the asset again at date 2 when $U = 1$ and stay put if $U = 0$. Thus, using the expression for the stock price in Case B1 of Step 2, speculators' expected profit in this case is:

$$\begin{aligned} & \theta \times \left[Pr[f_2 \in [-Q + \beta, Q - \beta]] \times \left(1 - \frac{1}{2}\right) + Pr[f_2 \in [Q - \beta, Q]] \times \left(1 - \frac{1}{2 - \theta}\right) + Pr[f_2 \in [Q, Q + \beta]] \times 0 \right] \\ &= \left[\frac{Q - \beta}{Q} \times \frac{1}{2} + \frac{\beta}{2Q} \times \frac{1 - \theta}{2 - \theta} \right] = \frac{\theta}{2Q} \times \left[\left(Q - \beta + \beta \frac{1 - \theta}{2 - \theta} \right) \right]. \end{aligned}$$

Case 2. If $f_1 \in [Q - \alpha, Q + \alpha]$, the stock price at date 1 is $\mu(1)$. Hence, in period 2 speculators buy if $U = 1$ (in which case $f_2 \in [-Q + \beta, Q + \beta]$; Case C1 in Step 2) and sell if $U = 0$ (in which case $f_2 \in [-Q - \beta, Q - \beta]$; Case A1 in Step 2). Their expected profit is then:

$$\begin{aligned} & \left\{ \theta \times \left[Pr[f_2 \in [-Q + \beta, Q - \beta]] \times \left(1 - \frac{1 + \theta}{2}\right) + Pr[f_2 \in [Q - \beta, Q + \beta]] \times 0 \right] + \right. \\ & \left. (1 - \theta) \times \left[Pr[f_2 \in [-Q - \beta, -Q + \beta]] \times 0 + Pr[f_2 \in [-Q + \beta, Q - \beta]] \times \left(\frac{1 + \theta}{2} - \frac{1}{2}\right) \right] \right\} \\ &= \left[\theta \times \left(\frac{Q - \beta}{Q} \times \frac{1 - \theta}{2} \right) + (1 - \theta) \times \left(\frac{Q - \beta}{Q} \times \frac{\theta}{2} \right) \right] = \frac{\theta}{2Q} \times (1 - \theta)(Q - \beta). \end{aligned}$$

Case 1 happens with probability $\left(\frac{Q - \alpha}{Q}\right)$ and Case 2 happens with probability $\frac{\alpha}{Q}$. Hence:

$$\pi_2(\alpha, \beta, 1) = \frac{\theta}{2Q^2} \times \left[(Q - \alpha) \left(Q - \beta + \beta \frac{1 - \theta}{2 - \theta} \right) + (1 - \theta)\alpha(Q - \beta) \right].$$

By symmetry, we have $\pi_2(\alpha, \beta, 0) = \pi_2(\alpha, \beta, 1)$. Thus:

$$\bar{\pi}_2(\alpha, \beta) = \frac{\theta}{2Q^2} \times \left[(Q - \alpha) \left(Q - \beta + \beta \frac{1 - \theta}{2 - \theta} \right) + (1 - \theta)\alpha(Q - \beta) \right] \quad \text{for } \beta \leq Q.$$

Now suppose that $Q \leq \beta \leq 2Q$. In this case, speculators obtain a positive expected profit only when $p_1 = 1/2$ (i.e., $f_1 \in [-Q + \alpha, Q - \alpha]$) and $U_1 = 1$ (see the last part of Step 2). Hence their expected profit when $S_1 = 1$ is:

$$\begin{aligned} \bar{\pi}_2(\alpha, \beta, 1) &= \theta \left(\frac{Q - \alpha}{Q} \right) \times \left(1 - \frac{1}{2 - \theta} \right) \times \left(\frac{2Q - \beta}{2Q} \right) \\ &= \theta \left(\frac{1 - \theta}{2 - \theta} \right) \left(\frac{Q - \alpha}{Q} \right) \left(\frac{2Q - \beta}{Q} \right). \end{aligned}$$

Proof of Corollary ??.. It is immediate from (??) that $\bar{\pi}_1(\alpha)$ increases with θ and decreases

with α . This proves part 1 of the corollary and the statement regarding α in the last part. Let:

$$G(x, y, \theta) = \theta \left[(1-x) \left(1 - \frac{1}{2-\theta}y \right) + (1-\theta)x(1-y) \right].$$

Observe that $\bar{\pi}_2(\alpha, \beta, \theta) = G(\alpha/Q, \beta/Q, \theta)/2$ when $\beta \leq Q$. Thus, when $\beta \leq Q$, the derivative of $\bar{\pi}_2$ with respect to θ has the same sign that the derivative of $G(x, y, \theta)$ with respect to θ for x, y, θ in $[0, 1]$. We have:

$$\begin{aligned} \frac{\partial G}{\partial \theta} &= (1-x) \left(1 - \frac{1}{2-\theta}y \right) + (1-\theta)x(1-y) + \theta \left[-\frac{1}{(2-\theta)^2}(1-x)y - x(1-y) \right], \\ &= 1-x - (1-x)y \left[\frac{\theta}{(2-\theta)^2} - \frac{1}{2-\theta} \right] + x(1-y)(1-2\theta), \\ &= 1-x - (1-x)y \frac{2}{(2-\theta)^2} + x(1-y)(1-2\theta), \\ &= 1-xy - 2 \left[(1-x)y \frac{1}{(2-\theta)^2} + x(1-y)\theta \right]. \end{aligned}$$

Clearly:

$$\frac{\partial^2 G}{\partial \theta^2} < 0.$$

Thus, $\frac{\partial G}{\partial \theta}$ decreases with θ on $[0, 1]$. Moreover,

$$\frac{\partial G}{\partial \theta} \Big|_{\theta=0} = 1 - \frac{y}{2} - \frac{xy}{2} = 1 - \frac{(1+x)y}{2} > 0,$$

and using the first line for the expression of $\frac{\partial G}{\partial \theta}$, we find:

$$\frac{\partial G}{\partial \theta} \Big|_{\theta=1} = (1-x)(1-y) - (1-x)y - x(1-y) = (1-x)(1-y) \left[1 - \frac{x}{1-x} - \frac{y}{1-y} \right].$$

If $\frac{x}{1-x} + \frac{y}{1-y} < 1$, $\frac{\partial G}{\partial \theta} \Big|_{\theta=1} > 0$. Hence, as $\frac{\partial G}{\partial \theta}$ decreases on in θ on $[0, 1]$ then

$$\frac{\partial G}{\partial \theta} > 0 \quad \forall \theta \in [0, 1].$$

If $\frac{x}{1-x} + \frac{y}{1-y} > 1$, $\frac{\partial G}{\partial \theta} \Big|_{\theta=1} < 0$. Hence, as $\frac{\partial G}{\partial \theta}$ decreases on in θ on $[0, 1]$, there is a unique $\theta(x, y)$ such that:

$$\frac{\partial G}{\partial \theta} > 0 \quad \forall \theta \in [0, \theta(x, y)] \quad \text{and} \quad \frac{\partial G}{\partial \theta} < 0 \quad \forall \theta \in [\theta(x, y), 1].$$

This unique $\theta(x, y)$ solves:

$$\frac{\partial G}{\partial \theta} = 0.$$

Parts 2 and 3 of the corollary follows defining $\Theta(\alpha, \beta) = \theta(\alpha/Q, \beta/Q)$.

When $Q \leq \beta \leq 2Q$, using the last part of Proposition ??, we obtain:

$$\frac{\partial \bar{\pi}_2}{\partial \theta} = \frac{(1-2\theta)(2-\theta) + \theta(1-\theta)}{(2-\theta)^2} = \frac{2-4\theta+\theta^2}{(2-\theta)^2} = \frac{(2+\sqrt{2}-\theta)(2-\sqrt{2}-\theta)}{(2-\theta)^2}.$$

This is positive for $\theta \in [0, 2-\sqrt{2}]$ and negative for $\theta \in [2-\sqrt{2}, 1]$.

Now consider the effect of α and β on $\bar{\pi}_2$. When $\beta \leq Q$, we have:

$$\frac{\partial \bar{\pi}_2(\alpha, \beta)}{\partial \alpha} = \frac{\theta}{2Q} \left[-\left(1 - \frac{1}{2-\theta} \frac{\beta}{Q}\right) + (1-\theta)\left(1 - \frac{\beta}{Q}\right) \right], = -\frac{\theta}{2Q} \left[\theta + \left(1-\theta - \frac{1}{2-\theta}\right) \frac{\beta}{Q} \right]. \quad (36)$$

If $1-\theta - \frac{1}{2-\theta} > 0$ then

$$\frac{\partial \bar{\pi}_2(\alpha, \beta)}{\partial \alpha} < 0.$$

If $1-\theta - \frac{1}{2-\theta} < 0$, then (??) implies that $\frac{\partial \bar{\pi}_2(\alpha, \beta)}{\partial \alpha}$ is maximal for $\beta = Q$. Hence:

$$\frac{\partial \bar{\pi}_2(\alpha, \beta)}{\partial \alpha} < \frac{\theta}{2Q} \left[-\theta + \left(\frac{1}{2-\theta} - (1-\theta)\right) \right] = -\frac{\theta}{2Q} \left[\frac{1-\theta}{2-\theta} \right] < 0.$$

We deduce that, when $\beta \leq Q$,

$$\frac{\partial \bar{\pi}_2(\alpha, \beta)}{\partial \beta} < 0.$$

This is also clearly the case when $Q \leq \beta \leq 2Q$ using (??). Finally, using (??) again, it is straightforward that $\bar{\pi}_2(\alpha, \beta)$ decreases with β .

Proof of Proposition ??.

The three first parts of the proposition follows directly from the text. For the last part, we need to compare $\beta^*(0)$ and $\beta^*(\alpha^*(0))$. As $\bar{\Delta} > 0$, we deduce that $\beta^*(\alpha^*(0)) = \alpha^*(0)$. Thus, using (??), we deduce that $\beta^*(\alpha^*(0))$ solves:

$$\Pi^F(\beta^*(\alpha^*(0)), \beta^*(\alpha^*(0))) = \bar{\pi}_1(\beta^*(\alpha^*(0))) + \bar{\pi}_2(\beta^*(\alpha^*(0)), \beta^*(\alpha^*(0))) = C_p.$$

As (a) $\Pi^F(\alpha, \beta)$ decreases in α and β , (b) $\Pi^F(0, 0) = \theta$, and (c) $\Pi^F(Q, Q) = 0$, we deduce that $0 \leq \beta^*(\alpha^*(0)) \leq Q$.

Now consider $\beta^*(0)$ given in (??). If $C_p \leq \frac{\theta(1-\theta)}{2(2-\theta)}$ then $\beta^*(0) \geq Q$. Thus, $\beta^*(0) \geq \beta^*(\alpha^*(0))$ in this case. Furthermore, for $\frac{\theta(1-\theta)}{2(2-\theta)} \leq C_p \leq \frac{\theta}{2}$, $\beta^*(0)$ decreases in C_p from Q to 0. Thus, there exists one value C_p^* , in $(\frac{\theta(1-\theta)}{2(2-\theta)}, \frac{\theta}{2})$, such that $\beta^*(0) > \beta^*(\alpha^*(0))$ iff $C_p < C_p^*$.

In the rest of the proof, we provide a closed form characterization of $\bar{\Delta}$, α^* , and β^* in the different cases considered in the proposition. We first characterize the threshold $\bar{\Delta}(\theta, C_p)$ in closed-form in the next lemma.

Lemma 1. Let $f(\theta) = \frac{(1-3\theta+\theta^2)}{(2-\theta)}$.

$$\text{If } \theta < (3 - \sqrt{5})/2, \text{ where } f(\theta) > 0, \bar{\Delta}(\theta, C_p) = \frac{\theta}{2} \left[\frac{1+f(\theta)}{2f(\theta)} - \sqrt{\left(\frac{1+f(\theta)}{2f(\theta)}\right)^2 - \frac{2C_p}{\theta f(\theta)}} \right],$$

$$\text{If } \theta = (3 - \sqrt{5})/2, \text{ where } f(\theta) = 0, \bar{\Delta}(\theta, C_p) = C_p,$$

$$\text{If } \theta > (3 - \sqrt{5})/2, \text{ where } f(\theta) < 0, \bar{\Delta}(\theta, C_p) = \frac{\theta}{2} \left[\frac{1+f(\theta)}{2f(\theta)} + \sqrt{\left(\frac{1+f(\theta)}{2f(\theta)}\right)^2 - \frac{2C_p}{\theta f(\theta)}} \right]$$

$$\text{or alternatively with } h(\theta) = -f(\theta) > 0, \bar{\Delta}(\theta, C_p) = \frac{\theta}{2} \left[\sqrt{\left(\frac{1-h(\theta)}{2h(\theta)}\right)^2 + \frac{2C_p}{\theta h(\theta)}} - \frac{1-h(\theta)}{2h(\theta)} \right]$$

Moreover, if $f(\theta) < 0$, $\bar{\Delta}(\theta, C_p)$ increases with θ :

$$\text{If } \theta > (3 - \sqrt{5})/2, \quad \frac{\partial \bar{\Delta}}{\partial \theta} > 0.$$

Proof of Lemma ??.

In equilibrium, when $\beta^* = \alpha^*$, we necessarily have $\beta^* \leq Q$ because $\alpha^* \leq Q$. We deduce from the last part of Proposition ?? that:

$$\bar{\pi}_2(\alpha^*(\Delta), \alpha^*(\Delta)) = \frac{\theta}{2} \left[\left(1 - \frac{\alpha^*(\Delta)}{Q}\right) \left(1 - \frac{1}{2-\theta} \frac{\alpha^*(\Delta)}{Q}\right) + (1-\theta) \frac{\alpha^*(\Delta)}{Q} \left(1 - \frac{\alpha^*(\Delta)}{Q}\right) \right]. \quad (37)$$

By definition, $\bar{\Delta}$ solves $\bar{\pi}_2(\alpha^*(\Delta), \alpha^*(\Delta)) = C_p$. As Δ affects $\bar{\pi}_2(\alpha^*(\Delta), \alpha^*(\Delta))$ only through its effect on α^* , we can solve for $\bar{\Delta}$ by first solving $\bar{\pi}_2(\alpha, \alpha) = C_p$ for $\alpha \in [0, Q]$ and then use the fact that $\alpha^*(\bar{\Delta}) = Q(1 - 2\bar{\Delta}/\theta)$ (see eq. ??) to obtain $\bar{\Delta}$. To this end, let $f(\theta) = 1 - \theta - \frac{1}{2-\theta}$ and:

$$P(x) = 1 - (1 - f(\theta))x - f(\theta)x^2 - 2C_p/\theta. \quad (38)$$

Using (??), we deduce that solving $\bar{\pi}_2(\alpha^*, \alpha^*) = C_p$ is equivalent to find x^* such that $P(x^*) = 0$

for $x^* \in [0, 1]$. Indeed, we have: $\alpha^* = Qx^*$. Observe that $1 - f(\theta) = \theta + 1/(2 - \theta) > 0$ and $1 + f(\theta) = 2 - \theta - 1/(2 - \theta) > 0$. Hence $f(\theta) \in [-1, 1]$. As $P(x)$ is quadratic, it has at most two roots. Now, as $P(0) = 1 - 2C_p/\theta > 0$ (as $C_p < \frac{\theta}{2}$) and $P(1) = -2C_p/\theta < 0$, we deduce that it has a unique root $x^* \in [0, 1]$.

Case 1. If $f(\theta) > 0$, $P(x)$ is a concave quadratic function. Hence, x^* is the unique positive root of $P(x)$. In this case the determinant of $P(x)$ is

$$\text{Det} = (1 - f(\theta))^2 + 4f(\theta)(1 - 2\frac{C_p}{\theta}) > 0,$$

and

$$x^* = \frac{\sqrt{(1 - f(\theta))^2 + 4f(\theta)(1 - 2\frac{C_p}{\theta})} - \frac{1 - f(\theta)}{2f(\theta)}}{2f(\theta)}.$$

Hence:

$$\begin{aligned} \alpha^*(\bar{\Delta}) &= Q \left(\frac{\sqrt{(1 - f(\theta))^2 + 4f(\theta)(1 - 2\frac{C_p}{\theta})}}{2f(\theta)} - \frac{1 - f(\theta)}{2f(\theta)} \right) \\ &= Q \left(\frac{\sqrt{(1 + f(\theta))^2 - 8f(\theta)\frac{C_p}{\theta}}}{2f(\theta)} - \frac{1 - f(\theta)}{2f(\theta)} \right) \\ &= Q \left(\sqrt{\left(\frac{1 + f(\theta)}{2f(\theta)}\right)^2 - 2\frac{C_p}{\theta f(\theta)}} - \frac{1 - f(\theta)}{2f(\theta)} \right) \end{aligned}$$

Finally, using the fact that $\alpha^*(\bar{\Delta}) = Q(1 - 2\bar{\Delta}/\theta)$, we deduce that:

$$\bar{\Delta} = \frac{\theta}{2} \left(\frac{1 + f(\theta)}{2f(\theta)} - \sqrt{\left(\frac{1 + f(\theta)}{2f(\theta)}\right)^2 - 2\frac{C_p}{\theta f(\theta)}} \right)$$

Case 2. If $f(\theta) = 0$, we have:

$$x^* = 1 - \frac{2C_p}{\theta},$$

and therefore, using $\alpha^* = Qx^*$ and $\alpha^*(\bar{\Delta}) = Q(1 - 2\bar{\Delta}/\theta)$, we deduce that:

$$\bar{\Delta} = C_p.$$

Case 3. If $f(\theta) < 0$, $P(x)$ is convex. Hence, it has two positive roots but its second root must be larger than 1 since x^* is the unique root in $[0, 1]$. Let $h(\theta) = -f(\theta)$. The determinant of

$P(x)$ is

$$Det = (1 + h(\theta))^2 - 4h(\theta)(1 - 2\frac{C_p}{\theta}) = (1 - h(\theta))^2 + 8h(\theta)\frac{C_p}{\theta} > 0,$$

and its two roots are

$$\frac{1 + h(\theta)}{2h(\theta)} \pm \frac{\sqrt{(1 - h(\theta))^2 + 8h(\theta)\frac{C_p}{\theta}}}{2h(\theta)}.$$

Choosing the smallest root and using the fact that $\alpha^* = Qx^*$, we deduce that:

$$\begin{aligned} \alpha^*(\bar{\Delta}) &= Q \left(\frac{1 + h(\theta)}{2h(\theta)} - \frac{\sqrt{(1 - h(\theta))^2 + 8h(\theta)\frac{C_p}{\theta}}}{2h(\theta)} \right), \\ &= Q \left(\frac{1 + h(\theta)}{2h(\theta)} - \sqrt{\left(\frac{1 - h(\theta)}{2h(\theta)}\right)^2 + 2\frac{C_p}{\theta h(\theta)}} \right). \end{aligned}$$

Finally, using the fact that $\alpha^*(\bar{\Delta}) = Q(1 - 2\bar{\Delta}/\theta)$, we deduce that:

$$\begin{aligned} \bar{\Delta} &= \frac{\theta}{2} \left(\sqrt{\left(\frac{1 - h(\theta)}{2h(\theta)}\right)^2 + 2\frac{C_p}{\theta h(\theta)}} - \frac{1 - h(\theta)}{2h(\theta)} \right) \\ &= \frac{\theta}{2} \left(\sqrt{\left(\frac{1 + f(\theta)}{2f(\theta)}\right)^2 - 2\frac{C_p}{\theta f(\theta)}} + \left(\frac{1 + f(\theta)}{2f(\theta)}\right) \right) \end{aligned}$$

Derivative of $\bar{\Delta}$ with θ . As $\alpha^*(\bar{\Delta}) = Q(1 - 2\bar{\Delta}/\theta)$, using (??), we deduce that $\bar{\Delta}$ solves:

$$\frac{\theta}{2} \left[1 - (1 - f(\theta)) \left(1 - \frac{2\bar{\Delta}}{\theta}\right) - f(\theta) \left(1 - \frac{2\bar{\Delta}}{\theta}\right)^2 \right] = \bar{\Delta} \left[1 + f(\theta) - 2\frac{f(\theta)}{\theta}\bar{\Delta} \right] = C_p.$$

The L.H.S of this equation increases with Δ . Furthermore, its derivative with respect to θ is:

$$\frac{\partial}{\partial \theta} \left[1 + f(\theta) - \frac{2f(\theta)}{\theta}\bar{\Delta} \right] = \frac{\partial f}{\partial \theta} \left(1 - \frac{2\bar{\Delta}}{\theta}\right) + \frac{2f(\theta)}{\theta^2}\bar{\Delta},$$

which is negative when $f(\theta) < 0$ because $\frac{\partial f}{\partial \theta} < 0$. Hence when $f(\theta) < 0$, $\bar{\Delta}$ increases with θ .

This is not necessarily the case when $f(\theta) > 0$. \square

Closed-form solutions for α^* and β^* .

Case 1. When $\Delta > \frac{\theta}{2}$, $\alpha^* = 0$ and $\beta^* = \beta^*(0)$, where $\beta^*(0)$ is given in (??).

Case 2. When $\frac{\theta}{2} > \Delta > \bar{\Delta}$, α^* is given by (??) and, as explained in the text, β^* solves $\Pi^S(\alpha^*, \beta^*) = 0$. To solve this equation, we must distinguish two cases. If $\frac{\theta(1-\theta)}{2(2-\theta)} \leq C_p \leq \frac{\theta}{2}$ then

$\beta^*(0) \leq Q$ and therefore $\beta^* \leq Q$ for $\alpha^* > 0$. Using (??), we deduce that β^* solves:

$$\frac{\theta}{2} \left[\left(1 - \frac{\alpha}{Q}\right) \left(1 - \frac{1}{2-\theta} \frac{\beta^*}{Q}\right) + (1-\theta) \frac{\alpha}{Q} \left(1 - \frac{\beta^*}{Q}\right) \right] = C_p,$$

which yields:

$$\beta^* = Q \frac{\theta - 2C_p - \theta^2 \frac{\alpha^*}{Q}}{\frac{\theta}{2-\theta} + \left(\theta - \theta^2 - \frac{\theta}{2-\theta}\right) \frac{\alpha^*}{Q}}, \quad (39)$$

where α^* is given by (??).

If $C_p \leq \frac{\theta(1-\theta)}{2(2-\theta)}$ then $Q \leq \beta^*(0) \leq 2Q$. Thus, there is a range of values for Δ such that $\beta^* \geq Q$. Using (??), we obtain that for these values, it must be the case that:

$$\beta^* = 2Q \left(1 - \left(\frac{2-\theta}{1-\theta}\right) \frac{C_p}{2\Delta}\right).$$

It is then easily checked that $\beta^* \geq Q$ for $\Delta \geq \left(\frac{2-\theta}{1-\theta}\right) C_p$. When $\Delta \leq \left(\frac{2-\theta}{1-\theta}\right) C_p$ then $\beta^* \leq Q$ and is given by (??).

Case 3. When $\Delta < \bar{\Delta}$, $\alpha^* = \beta^*$. Then, as explained in the text, β^* (or α^*) solves $\bar{\pi}_1(\beta^*, \beta^*) + \bar{\pi}_2(\beta^*, \beta^*) = C_F$. We deduce that in this case:

$$\begin{aligned} \text{If } \theta < (3 - \sqrt{5})/2, \quad \frac{\beta^*}{Q} &= \sqrt{\left(\frac{2+f(\theta)}{2f(\theta)}\right)^2 - 2\frac{C_F}{\theta f(\theta)} - \frac{2-f(\theta)}{2f(\theta)}}, \\ \text{If } \theta = (3 - \sqrt{5})/2, \quad \frac{\beta^*}{Q} &= 1 - \frac{C_F}{\theta}, \\ \text{If } \theta > (3 - \sqrt{5})/2, \quad \frac{\beta^*}{Q} &= \frac{2+h(\theta)}{2h(\theta)} - \sqrt{\left(\frac{2-h(\theta)}{2h(\theta)}\right)^2 + 2\frac{C_F}{\theta h(\theta)}} \end{aligned}$$

where $f(\theta) = 1 - \theta - \frac{1}{2-\theta}$ and $h(\theta) = -f(\theta)$.

Proof of Corollary ??. The expected absolute price change in period 1 is:

$$E(|p_1 - \frac{1}{2}|) = \frac{\theta\alpha^*}{2Q},$$

which increases in θ when α^* increases with θ . This is the case for $\Delta > \bar{\Delta}$, because then $\alpha^* = \text{Max}\{Q(1 - 2\frac{\Delta}{\theta}), 0\}$.

For $\Delta < \bar{\Delta}$, α^* solves:

$$\bar{\pi}_1(\alpha) + \bar{\pi}_2(\alpha, \alpha) = C_F$$

However we need to extract, from this equation, the variations of $\theta\alpha^*$ and not only α^* . Let's call $X = \theta\alpha/Q$. We can rewrite the previous equation as

$$h(X) = (\theta - X) \left(2 - \frac{X}{\theta(2 - \theta)} \right) + \frac{1 - \theta}{\theta} X(\theta - X) = 2C_F.$$

Since $\bar{\pi}_1(\alpha) + \bar{\pi}_2(\alpha, \alpha)$ decreases with α , we already know that h decreases with X . Now we want to study the variation of h with respect to θ . Then we compute the derivative of h with respect to θ ,

$$\frac{\partial h}{\partial \theta} = 2 - \frac{X}{\theta(2 - \theta)} + \frac{(2 - 2\theta)X(\theta - X)}{(\theta(2 - \theta))^2} + \frac{(1 - \theta)X}{\theta} - \frac{X(\theta - X)}{\theta^2}.$$

Let's notice that $0 < X < \theta$ because $\alpha < Q$, then $0 < \frac{X}{\theta} < 1$, $0 < 1 - \frac{X}{\theta} < 1$, and also $2 - \theta > 1$, hence we can slightly rearrange the term of the previous expression to show that it is positive,

$$\frac{\partial h}{\partial \theta} = \underbrace{1 - \frac{1}{2 - \theta} \frac{X}{\theta}}_{>0} + \underbrace{\frac{2(1 - \theta)X}{(2 - \theta)^2} \frac{X}{\theta} \left(1 - \frac{X}{\theta} \right)}_{>0} + \underbrace{\frac{(1 - \theta)X}{\theta}}_{>0} + \underbrace{1 - \frac{X}{\theta} \left(1 - \frac{X}{\theta} \right)}_{>0} > 0.$$

Then it shows that X^* , defined as the solution of $h(X) = 2C_F$, increases with θ .

Proof of Proposition ??.

If $\frac{\theta}{2} < C_p < \theta$, then $\beta^*(0) = 0$ because $\bar{\pi}_2(0, \beta) < C_p$ for all $\beta \geq 0$. As $\bar{\pi}_2(\alpha, \beta)$ decreases in both α and β , we deduce that $\bar{\pi}_2(\alpha, \beta) < C_p$ for all α and all β . Hence, there cannot be an equilibrium in which $\beta^* > \alpha^*$ as this would imply $\bar{\pi}_2(\alpha^*, \beta^*) = C_p$, which is impossible in this case.

Thus, if $\alpha^* > 0$ in equilibrium, it must be the case that $\beta^* = \alpha^*$ and α^* solves:

$$\Pi^F(\alpha^*, \alpha^*) = \pi_1(\alpha^*) + \pi_2(\alpha^*, \alpha^*) - C_p + \Delta = 0.$$

As $\Pi^F(\alpha, \beta)$ decreases in α and β and $\Pi^F(Q, Q) < C_p + \Delta$, a necessary and sufficient condition for $\alpha^* > 0$ is $\Pi^F(0, 0) > 0$. As $\bar{\pi}_1(0) = \bar{\pi}_2(0, 0) = \frac{\theta}{2}$, we have $\Pi^F(0, 0) = \bar{\pi}_1(0) + \bar{\pi}_2(0, 0) - (C_p + \Delta) = \theta - (C_p + \Delta)$, we deduce that $\alpha^* > 0$ iff $\theta < C_p + \Delta$. Otherwise $\beta^* = \alpha^* = 0$.

When $\alpha^* > 0$, it solves $\Pi^F(\alpha^*, \alpha^*) = 0$. This solution is given in Case 3 of the last part of the proof of Proposition ??. Finally, the last part of Proposition ?? follows directly from the fact that $\Pi^F(\alpha, \alpha)$ decreases in Δ .

Proof of Corollary ??. Using the first parts of Propositions ?? and ??, we deduce that:

$$x_1 = \mathbb{I}_{S_1=1} - \mathbb{I}_{S_1=0}, \text{ with } S_1 = U \times V + (1 - U) \times \epsilon, \quad (40)$$

$$x_2 = U \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1 - U) \times [\mathbb{I}_{p_1=(1-\theta)/2} - \mathbb{I}_{p_1=(1+\theta)/2}], \quad (41)$$

where \mathbb{I} denotes the indicator function, which is one when the statement in brackets holds. As $\mathbb{E}[x_1] = \mathbb{E}[x_2] = 0$, we deduce from (??) and (??) that:

$$\begin{aligned} Cov(x_1, x_2) &= \mathbb{E}[x_1 x_2] = \frac{1}{2} \mathbb{E}[x_2 | S_1 = 1] - \frac{1}{2} \mathbb{E}[x_2 | S_1 = 0], \\ &= \frac{\theta}{2} \mathbb{E}[x_2 | V = 1, U = 1] + \frac{1}{2} (1 - \theta) \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | \epsilon = 1, U = 0, p_1 = \frac{1 + \theta}{2} \right], \\ &\quad - \frac{\theta}{2} \mathbb{E}[x_2 | V = 0, U = 1] - \frac{1}{2} (1 - \theta) \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | \epsilon = 0, U = 0, p_1 = \frac{1 - \theta}{2} \right], \\ &= \theta - (1 - \theta) \frac{\alpha^*}{Q}. \end{aligned}$$

As α^* declines with Δ , we deduce that $Cov(x_1, x_2)$ decreases with Δ . When $\Delta > \bar{\Delta}$, we have: $\alpha^*(\Delta) = Q(1 - 2\Delta/\theta)$ (see eq. ??). Hence, the covariance between x_1 and x_2 is:

$$Cov(x_1, x_2) = \theta - (1 - \theta) \left(1 - \frac{2\Delta}{\theta} \right) = 2\theta - (1 + 2\Delta) + \frac{2\Delta}{\theta}.$$

After some algebra, we deduce that $Cov(x_1, x_2) < 0$ if and only if $\theta \in [\theta_1(\Delta), \theta_2(\Delta)]$ where

$$0 < \theta_1(\Delta) = \frac{1 + 2\Delta - \sqrt{(1 + 2\Delta)^2 - 16\Delta}}{4} < \theta_2(\Delta) = \frac{1 + 2\Delta + \sqrt{(1 + 2\Delta)^2 - 16\Delta}}{4} < 1.$$

Proof of Corollary ??. Using the second part of Proposition ?? and the first part of Proposition ??, we deduce that:

$$p_1 = \frac{1}{2} + \frac{\theta}{2} \mathbb{I}_{f_1 > Q - \alpha^*} - \frac{\theta}{2} \mathbb{I}_{f_1 < -Q + \alpha^*} \quad (42)$$

$$x_2 = U \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1 - U) \times [\mathbb{I}_{p_1=(1-\theta)/2} - \mathbb{I}_{p_1=(1+\theta)/2}]. \quad (43)$$

As $\mathbb{E}[x_2] = 0$ and $\mathbb{E}[p_1] = 1/2$, we deduce from (??) and (??) that:

$$\begin{aligned}
Cov(p_1, x_2) &= \mathbb{E}[(p_1 - 1/2)x_2] = \frac{\theta}{2} \left\{ \frac{1}{2} \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | S_1 = 1, p_1 = \frac{1+\theta}{2} \right] - \frac{1}{2} \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | S_1 = 0, p_1 = \frac{1-\theta}{2} \right] \right\} \\
&= \frac{\theta^2}{4} \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | V = 1, U = 1, p_1 = \frac{1+\theta}{2} \right] + \frac{\theta(1-\theta)}{4} \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | \epsilon = 1, U = 0, p_1 = \frac{1+\theta}{2} \right] \\
&\quad - \frac{\theta^2}{4} \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | V = 0, U = 1, p_1 = \frac{1-\theta}{2} \right] - \frac{\theta(1-\theta)}{4} \frac{\alpha^*}{Q} \mathbb{E} \left[x_2 | \epsilon = 0, U = 0, p_1 = \frac{1-\theta}{2} \right] \\
&= \theta(2\theta - 1) \frac{\alpha^*}{2Q}.
\end{aligned}$$

Proof of Corollary ??. First, consider $\mathcal{E}_1(\Delta, C_p) = E[(\tilde{V} - p_1)^2]$. We have:

$$\mathcal{E}_1(\Delta, C_p) = E(E[(\tilde{V} - p_1)^2]) = E[(\tilde{V} - p_1)^2 | f_1].$$

Using Proposition ?? and the fact that $\alpha^* < Q$, we deduce, after some algebra, that:

$$E[(\tilde{V} - P_1)^2] = \frac{1}{4} \left[1 - \theta^2 \frac{\alpha}{Q} \right] = \frac{1}{4} - \frac{\theta}{2} \left(\frac{\theta}{2} - \bar{\pi}_1(\alpha^*) \right). \quad (44)$$

Second, consider $\mathcal{E}_2(\Delta, C_p) = E[(\tilde{V} - p_2)^2]$. We have:

$$\mathcal{E}_2(\Delta, C_p) = E(E[(\tilde{V} - p_2)^2]) = E[(\tilde{V} - p_1)^2 | f_1, f_2].$$

Using this observation, Proposition ??, and Proposition ??, we obtain, after straightforward but tedious, calculations, we obtain:¹⁸

$$\mathcal{E}_2(\Delta, C_p) = E[(\tilde{V} - P_2)^2] = \frac{1}{4} + \frac{1}{4} \left[-\theta^2 \frac{\alpha}{Q} - \frac{\theta}{2-\theta} \frac{\beta}{Q} + \theta \left(\frac{1}{2-\theta} - 1 + \theta \right) \frac{\alpha\beta}{Q^2} \right] \quad (45)$$

Now, for $\beta \leq Q$, we have (from Proposition ??):

$$\begin{aligned}
\bar{\pi}_2(\alpha, \beta) &= \frac{\theta}{2} \times \left[\left(1 - \frac{\alpha}{Q}\right) \times \left(1 - \frac{1}{2-\theta} \frac{\beta}{Q}\right) + (1-\theta) \frac{\alpha}{Q} \left(1 - \frac{\beta}{Q}\right) \right] \\
&= \frac{\theta}{2} \times \left[1 - \theta \frac{\alpha}{Q} - \frac{1}{2-\theta} \frac{\beta}{Q} + \left(\frac{1}{2-\theta} - 1 + \theta \right) \frac{\alpha\beta}{Q^2} \right]
\end{aligned}$$

Hence, we deduce that:

$$\mathcal{E}_2(\Delta, C_p) = \frac{1}{4} - \frac{1}{2} \left(\frac{\theta}{2} - \bar{\pi}_2(\alpha^*, \beta^*) \right). \quad (46)$$

¹⁸Detailed derivation of this equation is available upon request.

Then

$$\begin{aligned}
E[(\tilde{V} - P_2)^2] &= \frac{1}{2} \frac{\alpha(Q - \beta)}{Q^2} \times \frac{1 - \theta}{2} \frac{1 + \theta}{2} + \frac{1 - \theta}{2} \frac{\alpha\beta}{Q^2} \times \frac{1}{4} \\
&+ \frac{1 - \theta}{2} \frac{\alpha\beta}{Q^2} \times \frac{1}{4} + \frac{1}{2} \frac{\alpha(Q - \beta)}{Q^2} \times \frac{1 - \theta}{2} \frac{1 + \theta}{2} \\
&+ \frac{2 - \theta}{4} \frac{(Q - \alpha)\beta}{Q^2} \times \frac{1 - \theta}{2 - \theta} \frac{1}{2 - \theta} \\
&+ \frac{(Q - \alpha)(Q - \beta)}{Q^2} \frac{1}{4} + \frac{2 - \theta}{4} \frac{(Q - \alpha)\beta}{Q^2} \times \frac{1 - \theta}{2 - \theta} \frac{1}{2 - \theta} \\
&= \frac{1}{4} \frac{\alpha}{Q} \left[(1 - \theta)(1 + \theta) \frac{Q - \beta}{Q} + (1 - \theta) \frac{\beta}{Q} \right] + \frac{1}{4} \frac{Q - \alpha}{Q} \left[2 \frac{1 - \theta}{2 - \theta} \frac{\beta}{Q} + \frac{Q - \beta}{Q} \right]
\end{aligned}$$

Proof of Corollary ??. When $\Delta > \bar{\Delta}$, we have $\alpha^*(\Delta) = Q(1 - 2\frac{\Delta}{\theta})$. Hence, using (??), the likelihood of a price reversal is:

$$p_{Reversal}(\Delta, \theta) = (1 - \theta) \left(1 - \frac{2\Delta}{\theta} \right).$$

Thus:

$$\frac{\partial p_{Reversal}}{\partial \theta} = -1 + \frac{2\Delta}{\theta^2},$$

which is positive if and only if $\theta < \sqrt{2\Delta}$.

Proof of Corollary ??. When $\Delta < \bar{\Delta}$, all speculators are fast: $\beta^* = \alpha^*$. Hence, the mass of speculators increases when Δ decreases both at dates 1 and 2 (2nd part of Proposition ??).

Then it follows from (??) that $p_{quick\ Reversal}(\Delta, \theta)$ is larger when Δ is smaller.

Appendix B

To compute the likelihood of a quick price reversal when θ is stochastic, we just need to determine α^* and β^* in this case (see equation (??)). The analysis is identical to that in Section ?? but the closed-form expressions for $\bar{\Delta}$, α^* , and β^* . We outline the derivation of these expressions in this appendix.

When participants make their trading decisions at dates 1 and 2, they observe θ . Hence, Propositions ?? and ?? are unchanged. In particular, for a given θ , speculators' expected profit before observing the realizations of their signals are unchanged. Thus,

$$E[\bar{\pi}_1(\alpha)] = \frac{\bar{\theta}}{2} \text{Max}\left\{ \frac{Q - \alpha}{Q}, 0 \right\}.$$

It follows that $\alpha^* > 0$ for $\Delta < \frac{\theta}{2}$ and as long as $\beta^* > \alpha^*$, we have:

$$\alpha^*(\Delta) = Q \left(1 - 2 \frac{\Delta}{\theta} \right).$$

As Δ decreases, β^* decreases as in the analysis of Section ?? and there exists one threshold $\bar{\Delta}^r$ such that: $\beta^* = \alpha^*$. At this threshold, we necessarily have $\beta^* \leq Q$. In this case, speculators' ex-ante (at date 0) expected profit at date 2 is:

$$E[\pi_2(\alpha, \beta)] = \frac{1}{2} \left[\left(1 - \frac{\alpha}{Q} \right) \left(E[\theta] - E \left[\frac{\theta}{2-\theta} \right] \frac{\beta}{Q} \right) + E[\theta(1-\theta)] \frac{\alpha}{Q} \left(1 - \frac{\beta}{Q} \right) \right].$$

Observe that (i) $E[\pi_2(\alpha, \beta)]$ decreases in α and β , (ii) $E[\pi_2(0, 0)] = \frac{\bar{\theta}}{2}$, (iii) $E[\pi_2(Q, Q)] = 0$, and (iv) $E[\pi_2(\alpha, \alpha)] \geq 0$. Thus, there is a unique $\bar{\Delta}$ such that $E[\pi_2(\alpha^*(\bar{\Delta}), \alpha^*(\bar{\Delta}))] = C_p$. Proceeding exactly as in the proof of Lemma ?? in the proof of Proposition ??, we deduce that the threshold $\bar{\Delta}^r$ is:

$$\text{If } F(\theta) > 0, \bar{\Delta}^r = \frac{E[\theta]}{2} \left[\frac{1 + F(\theta)}{2F(\theta)} - \sqrt{\left(\frac{1 + F(\theta)}{2F(\theta)} \right)^2 - \frac{2C_p}{E[\theta]F(\theta)}} \right],$$

$$\text{If } F(\theta) = 0, \bar{\Delta}^r = C_p,$$

$$\text{If } F(\theta) < 0, \bar{\Delta}^r = \frac{E[\theta]}{2} \left[\sqrt{\left(\frac{1 - H(\theta)}{2H(\theta)} \right)^2 + \frac{2C_p}{E[\theta]H(\theta)}} - \frac{1 - H(\theta)}{2H(\theta)} \right].$$

where:

$$F(\theta) = \frac{E[\theta f(\theta)]}{E[\theta]} = \frac{E[\theta] - E[\theta^2] - E \left[\frac{\theta}{2-\theta} \right]}{E[\theta]}, \quad (47)$$

and $H(\theta) = -F(\theta) > 0$. The closed-form expressions for α^* and β^* are then as follows. When $\Delta > \frac{\bar{\theta}}{2}$:

$$\alpha^* = 0, \beta^* = Q \left(\frac{E[\theta] - 2C_p}{E \left[\frac{\theta}{2-\theta} \right]} \right). \quad (48)$$

When $\frac{\bar{\theta}}{2} > \Delta > \bar{\Delta}$, we have:

$$\alpha^* = Q \left(1 - \frac{2\Delta}{E[\theta]} \right), \beta^* = Q \frac{E[\theta] - 2C_p - E[\theta^2] \frac{\alpha^*}{Q}}{E \left[\frac{\theta}{2-\theta} \right] + \left(E[\theta] - E[\theta^2] - E \left[\frac{\theta}{2-\theta} \right] \right) \frac{\alpha^*}{Q}}. \quad (49)$$

When $\Delta < \bar{\Delta}$, $\alpha^* = \beta^*$ and β^* solves $E[\pi_1(\beta^*, \beta^*) + \pi_2(\beta^*, \beta^*)] = C_F$, which gives

$$\begin{aligned} \text{If } F(\theta) > 0, \beta^* &= Q \left(\sqrt{\left(\frac{2 + F(\theta)}{2F(\theta)} \right)^2 - 2 \frac{C_F}{E[\theta]F(\theta)} - \frac{2 - F(\theta)}{2F(\theta)}} \right), \\ \text{If } F(\theta) = 0, \beta^* &= Q \left(1 - \frac{C_F}{E[\theta]} \right), \\ \text{If } F(\theta) < 0, \beta^* &= Q \left(\frac{2 + H(\theta)}{2H(\theta)} - \sqrt{\left(\frac{2 - H(\theta)}{2H(\theta)} \right)^2 + 2 \frac{C_F}{E[\theta]H(\theta)}} \right), \end{aligned}$$

where $F(\theta)$ is given in (??) and $H(\theta) = -F(\theta) > 0$.

Numerical Simulation. To compute numerically the likelihood of a quick reversal when θ has cumulative distribution $G(\cdot)$, we must compute $F(\theta)$, which depends on different moments of the distribution for θ . Let denote the n th moment of θ by:

$$M_n = E[\theta^n]$$

Hence $E[\theta^1] = M_1$, $E[\theta^2] = M_2$, and:

$$E\left[\frac{\theta}{2-\theta}\right] = E\left[\frac{\theta}{2} \frac{1}{1-\frac{\theta}{2}}\right] = E\left[\frac{\theta}{2} \sum_{i=0}^{\infty} \frac{\theta^i}{2^i}\right] = E\left[\sum_{i=1}^{\infty} \frac{\theta^i}{2^i}\right] = \sum_{i=1}^{\infty} \frac{M_i}{2^i} = \Sigma$$

Then we deduce that:

$$F(\theta) = \frac{M_1 - M_2 - \Sigma}{M_1}$$

For numerical simulations, we approximate the infinite sum Σ by a truncation at rank N :

$$\Sigma_N = \sum_{i=1}^N \frac{M_i}{2^i}.$$

When $\theta = X^\lambda$ and X is a random variable drawn from the uniform distribution on $[0, 1]$, we have:

$$\begin{aligned} \bar{\theta} = M_1 = E[\theta] &= \frac{1}{\lambda + 1} \\ M_n = E[\theta^n] = E[X^{\alpha n}] &= \frac{1}{n\lambda + 1} = \frac{1}{n \left(\frac{1}{\bar{\theta}} - 1 \right) + 1} = \frac{\bar{\theta}}{n - (n-1)\bar{\theta}}. \end{aligned}$$

Hence, all moments are only dependent on $\bar{\theta}$ and we can easily compute numerically the likelihood of a quick reversal using Σ_N as an approximation for Σ .

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