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DOCUMENT  
DE TRAVAIL  
N° 233

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REGIONAL FISCAL POLICIES IN A FEDERATION**

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# Monetary Rules and the Spillover of Regional Fiscal Policies in a Federation\*

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\*We are grateful to the NSF and the CNRS for financial support. We are especially grateful to Michel Guillard and Cecilia Garcia-Penalosa for their comments, as well as to seminar participants at the University of Texas, the Bank of Canada, the Federal Reserve Bank of Cleveland, the Federal Reserve Bank of Dallas, the Federal Reserve Bank of Kansas City, and T2M 2009 (Strasbourg) for comments and suggestions.

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**Abstract:** This paper studies the effects of monetary policy rules in a fiscal federation, such as the European Union. The focus of the analysis is the interaction between the fiscal policy of member countries (regions) and the monetary authority. Each of the countries structures its fiscal policy (spending and taxes) with the interests of its citizens in mind. Ricardian equivalence does not hold due to the presence of monetary frictions, modeled here as reserve requirements. When capital markets are integrated, the fiscal policy of one country influences equilibrium wages and interest rates. Under certain rules, monetary policy may respond to the price variations induced by regional fiscal policies. Depending on the type of rule it adopts, interventions by the monetary authority affect the magnitude and nature of the spillover from regional fiscal policy.

**Keywords:** Monetary Union, Inflation tax, Seigniorage, monetary rules, public debt.

**JEL classifications:** E31, E42, E58, E62.

**Résumé :** Nous étudions les effets de règles de politique monétaires dans une fédération, comme l'union européenne. Nous nous intéressons en particulier à l'interaction entre les politiques fiscales des pays membres et l'autorité monétaire. Chaque pays conçoit sa politique budgétaire et fiscale en se préoccupant des intérêts de ses résidents. L'équivalence ricardienne ne tient pas en présence de frictions monétaires comme les réserves obligatoires. Quand les marchés financiers sont intégrés, la politique fiscale d'un pays influence les salaires et le taux d'intérêt d'équilibre. Sous certaines règles, la politique monétaire peut réagir à des variations de prix provoquées par des autorités fiscales nationales. Selon le type de règle adoptée, les interventions par la banque centrale affectent l'ampleur et la nature des externalités transnationales créées par les politiques fiscales nationales.

**Mots-clés:** Union monétaire, taxe inflationniste, seignuriage, dette publique, règle monétaire.

**JEL classifications:** E31, E42, E58, E62.

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## Non-technical summary.

This paper studies the design of monetary policy in a fiscal federation, formed by a number of countries sharing a common currency. The focus of the analysis is the interaction between the fiscal policy of member countries and the monetary rule chosen by the common central bank.

Within any federation, there exist transnational spillovers, if only because the interest rate and the area-wide inflation rate are common to all members of the federation. Since these spillovers are affected by the fiscal policies enforced by member countries, the fiscal authorities are tempted to spread their tax burden onto others. Excluding bailouts, these linkages arise in two forms. First, the debt policies of large regions may impact equilibrium quantities and prices, which we term a “fiscal spillover”. Second, depending on the monetary rule in place, debt policy may elicit a response by the monetary authority. If, for example, the central bank responds to variations in prices, then fiscal spillovers create a dependence of monetary policy on regional fiscal policy. As a consequence, the choice of monetary policy rules will impact on the conduct and effects of fiscal policy by governments within a federation. Their interaction will affect the macro dynamics of the entire zone and of each member country.

This paper is partly motivated by recent experience within the European Monetary Union. This is a leading example of a federation in which member states conduct independent fiscal policies yet there is a single central bank. Within the EMU, the direct bailout of a governmental authority by the ECB is explicitly forbidden. Instead, any accommodation of regional debt in the EMU would occur through the “normal operations” of the European Central Bank.

We explore these issues in a multi-region overlapping generations model with money and capital. Each of the regions (countries) structures its fiscal policy (spending and taxes) with the interests of its citizens in mind. Each region perceives a gain to shifting its tax burden onto other federation members.

Whether the fiscal policy of one region can influence the welfare of agents in other regions depends in part on the source of money demand in the economy. In a frictionless environment in which money earns the same return as capital and bonds, if the monetary rule does not answer to the level of national debts, the tax and spending of one region is completely irrelevant due to standard Ricardian type arguments. In this case, the fiscal policy of one region has no effect on individuals in other regions.

But, if the demand for money reflects frictions, which appear as reserve requirements in our model, then the fiscal policy of one region can impact equilibrium variables. In particular, the choice of the debt level in one region will influence aggregate capital holdings since Ricardian equivalence is broken and thus the equilibrium interest rates and wages.

In this situation, monetary policy matters for the magnitude and extent of the fiscal spillovers. Depending on the type of rule it adopts, the monetary authority will be induced to respond to national debt. In this way, in addition to influencing the equilibrium values of interest rates and wages directly, the fiscal policy of a regional government may impact these variables indirectly by inducing a response by the monetary authority.

To study the interactions between regional fiscal policy and the monetary policy response, we highlight a comparison of rules which fix money growth and those which peg an interest rate. Intuitively, apparently

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weak monetary rules such as pegging the interest rate would facilitate the redistribution of tax burdens through the monetization of the regional debt. In doing so, it would tax the money holdings of agents in the other region. In contrast, a strong monetary authority would fix the growth rate of the money supply and thus be immune to fiscal pressures. Our analysis refines this intuition and indicates that some forms of interest pegs insulate rather than expose agents in other regions. We also use our framework to highlight the interaction between the effects of monetary policy and asset market participation.

# 1 Introduction

This paper studies the design of monetary policy in a fiscal federation. The focus of the analysis is the interaction between the fiscal policy of member countries and the monetary rule chosen by the common central bank.

Within any federation, member countries have a desire to spread their tax burden onto others. This might arise through a bailout of regional obligations by a central entity, either a Treasury, as in Cooper, Peled and Kempf (2008), or a monetary authority, as in Cooper, Peled and Kempf (2007).

Beyond these explicit bailouts, there are more subtle channels of interactions across countries within a federation. These linkages arise in two forms. First, the debt policies of large regions may impact on the equilibrium prices. Second, depending on the monetary rule in place, debt policy may elicit a response by the monetary authority. This response might not be termed a “bailout” since it appears to be part of the normal operating procedure of the central bank. Nonetheless, for some monetary rules, the response of the central bank to variations in prices that can be influenced by the fiscal policy of regional governments can have effects that are effectively a “bailout”.

Thus our analysis highlights the interplay between fiscal policy and monetary rules. The choice of monetary policy rules will impact on the conduct and effects of fiscal policy by governments within a federation.

This paper is partly motivated by recent experience within the European Monetary Union. This is a leading example of a federation in which member states conduct independent fiscal policies yet there is a single central bank. Within the EMU, there have been numerous attempts to restrict fiscal policy at the national level. The costs of these restrictions arise from the limits on tax smoothing as well as the inability of member states to respond to country specific shocks through fiscal policy. The gains to these restrictions are less clear.

As argued in Cooper, Kempf and Peled (2008), a central fiscal entity may have an incentive to pay the debt obligations of regional governments. If so, there may be a rationale for limits on deficits. But that analysis does not pertain directly to the EMU in which there is no supra national fiscal entity. Instead, any bailout of regional debt in the EMU would occur through the operations of the European Central Bank. Thus we are interested in how the choice of monetary policy rule by a monetary authority, such as the ECB, impacts on fiscal policy by individual countries and thus on the need for fiscal restrictions.

We explore these issues in a multi-region overlapping generations model with money and capital. Each of the regions (countries) structures its fiscal policy (spending and taxes) with the interests of its citizens in mind. Each region perceives a gain to shifting its tax burden onto other federation members. The main issue addressed in this paper is how the choice of monetary policy facilitates the shifting of regional tax burdens.

Whether the fiscal policy of one region can influence the welfare of agents in other regions depends in part on the source of money demand in the economy. In a frictionless environment in which money earns the same return as capital and bonds, the tax and spending of one region is completely irrelevant due to standard Ricardian type arguments. In this case, the fiscal policy of one region has no effect on individuals in other regions.

But, if the demand for money reflects frictions, which appear as reserve requirements in our model, then

the fiscal policy of one region can impact equilibrium variables. We call these interactions *fiscal spillovers*. In particular, the choice of the debt level in one region will influence aggregate capital holdings and thus the equilibrium interest rates and wages.

In this situation, monetary policy matters for the magnitude and extent of the fiscal spillovers. Depending on the type of rule it adopts, the monetary authority can facilitate this redistribution of the tax burden. That is, in addition to influencing the equilibrium values of interest rates and wages, the fiscal policy of a region may induce a response by the monetary authority.

To study the interactions between regional fiscal policy and the monetary policy response, we highlight a comparison of rules which fix money growth and those which peg an interest rate.<sup>1</sup> Intuitively, one might conjecture that apparently weak monetary rules such as pegging the interest rate would facilitate the redistribution of tax burdens. If a region runs a deficit, selling its debt will impact interest rates. A monetary authority striving to peg interest rates may be induced to monetize the regional debt. In doing so, it would tax the money holdings of other agents. In contrast, a strong monetary authority would fix the growth rate of the money supply and thus be immune to fiscal pressures.

In fact, this intuition is incomplete in a couple of dimensions. First, there are spillovers even with fixed money growth rules. These arise through the dependence of equilibrium interest rates and wages on the aggregate capital stock.<sup>2</sup> Regional tax policy affects saving and thus the capital holdings of agents in that region. When regions are large, the fiscal policy of one region can influence wages and interest rates thus the choices and utility levels of agents in other regions.

Second, by pegging an interest rate, the monetary authority may in fact insulate the agents in one region from inflation and interest rate movements induced by the fiscal policy in another region. The key to the insulation is that the rate the monetary authority is pegging is the same as the return on agents' portfolios. So by pegging an interest rate, the link between the level of debt in one region and the return earned by households in other regions is broken. In this case, an apparently accommodative monetary rule is able to reduce the spillover effects of regional fiscal policy.<sup>3</sup> Our results highlight this insulation effect and then discuss conditions when it fails.

We also use our framework to highlight the interaction between the effects of monetary policy and asset market participation. If some agents do not save through intermediaries, then the insulation property of a fixed interest rate rule disappears and monetary policy has distribution effects across agents and regions.

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<sup>1</sup>Carlstrom and Fuerst (1995) also compare interest rate and money growth rules in the presence of cash-in-advance and portfolio restrictions. They do not study how these rules can facilitate fiscal spillovers. As in Carlstrom and Fuerst (1995), these two extreme policies meant to illustrate the potential for interaction across the two regions.

<sup>2</sup>This interaction is highlighted in Gaspar (2007) and is key to the comparison of cooperative and non-cooperative equilibria in Kehoe (1987).

<sup>3</sup>To be clear, this is not a statement about welfare, simply a counting of channels of influence.



## 2 Multi-Region Model

We study a multi-region (multi-country) infinite overlapping generations model with three stores of value: capital, money and bonds.<sup>4</sup> Agents live for two periods in one of two regions,  $i = 1, 2$ . The size of each generation is normalized to equal 1 with a fraction  $\eta^1$  of the agents living in region 1.

As our focus is on the interaction between the regions and the choice of monetary rules, we study the unique monetary steady state of the overlapping generations model. Thus the presentation of the basic model ignores time subscripts.

### 2.1 Households

Households are endowed with a unit of time in youth which is inelastically supplied to the labor market in return for a real wage  $\omega$ . Consumption occurs in both youth and old age. Hence, households consume a portion of their wage and save the rest. There are three stores of value: fiat money, loans to firms and government debt.

The household in region  $i$  receives a transfer  $g^i$  from its regional government in the first period and pays a tax of  $\tau_t^i$  in period  $t = y, o$  for  $i = 1, 2$  where  $t = y$  denotes youth and  $t = o$  denotes old age. The household in region  $i$  chooses how much to save, denoted  $s^i$  to solve

$$\max_s u(c_y^i) + v(c_o^i) \quad (1)$$

where  $c_y^i = w + g^i - \tau_y^i - s$  and  $c_o^i = sR - \tau_o^i$ . The real rate of return is denoted  $R$ . The first-order condition for the household is

$$u'(c_y^i) = Rv'(c_o^i). \quad (2)$$

Here  $R$  is interpreted as the rate of return offered by competitive intermediaries, described below. These intermediaries are open to all agents regardless of region (country). Thus financial markets are fully integrated.

Let  $s(\omega, R, \tau^y, \tau^o)$  denote the savings function of a household. We assume  $s_R(\omega, R, \tau^y, \tau^o) > 0$  so that substitution effects dominate.

### 2.2 Firms

Firms in both regions have access to the same constant returns to scale technology which converts labor ( $L$ ) and capital ( $K$ ) into the single consumption good:  $Y = F(K, L)$ . Firms maximize profits of  $F(K, L) - \omega L - rK$  leading to  $\omega = F_L(K, L)$  and  $r = F_K(K, L)$ . If  $k \equiv \frac{K}{L}$  is the per worker capital stock, then these first order conditions become  $\omega = f(k) - kf'(k)$  and  $r = f'(k)$  where  $f(k) = F(k, 1)$ . Capital fully depreciates in the production process.

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<sup>4</sup>The structure is similar to that in Kehoe (1987) except that we consider a monetary economy and allow regional governments to run deficits.

## 2.3 Regional Governments

The government in region 1, denoted  $RG^1$ , is active. It transfers an exogenous amount  $g^1$  to young agents of each generation and levy taxes on young and old agents. In addition, it receives a real transfer from the central bank of  $T^1$ . Looking at the region 1 government budget constraint from the perspective of a generation,

$$b = g^1 - \tau_y^1, \quad br = \tau_o^1 + T^1. \quad (3)$$

Here the region 1 government must pay the same return on its debt,  $r$ , paid to lenders by firms.

We summarize fiscal policy for region 1 by the amount of debt it issues. Once  $b$  is determined, taxes in the two periods come directly from (3) since  $g^1$  is given.

Note that we are forcing budget balance across time for each generation. In effect, each generation is represented by its own fiscal authority without interactions in fiscal policy across generations within a period.

Region 2 does not make transfers to young agents,  $g^2 = 0$ . However, depending on the nature of operations of the central bank, the region 2 government may receive transfers, denoted  $T^2$ , which are then rebated to old agents,  $\tau_o^2 = -T^2$ .

## 2.4 Intermediaries

There are intermediaries who provide the link between households and assets (loans to firms and government debt) and these intermediaries are subject to a reserve requirement. This reserve requirement creates a demand for money even when the return on bonds and loans exceeds the return on money.<sup>5</sup>

The intermediaries take in total deposit,  $S$ , and lend them to firms,  $k$  and to the region 1 government. There is a reserve requirement that a fraction  $\lambda$  of the deposit must be held as money. Hence  $\frac{M}{p} = \lambda S$  and  $b + k = (1 - \lambda)S$ . Thus, the return on deposits  $R$  satisfies

$$R = r(1 - \lambda) + \frac{\lambda}{\pi} \quad (4)$$

where  $r$  is, as above, the rental rate on capital, and  $\pi$  is 1 plus the inflation rate. Government bonds, as they compete with loans in the portfolio of the intermediary, must also have a return of  $r$ .

## 2.5 Central Bank

The Central Bank ( $CB$ ) controls the supply of money. Changes in the supply of money are brought about through transfers to the region 1 government and thus to agents in that region where  $\phi^i$  is the fraction of the transfer given to region  $i$ .<sup>6</sup>

Let  $T(k, b)$  be the transfer function of the  $CB$  where  $k$  is the per capita capital stock in the economy and  $b$  is the per capita level of region 1 debt, where  $b = \eta^1 b$ .<sup>7</sup> The transfer  $T(k, b)$  is per capita in the federation.

<sup>5</sup>The introduction of a reserve requirement follows Smith (1994).

<sup>6</sup>We do not consider open market operations. In some settings, as discussed in Smith (1994) these two forms of monetary operations may not be equivalent.

<sup>7</sup>In the analysis, we use  $b$  when referring to the debt (per member of region 1) of a regional government and use  $b$  as the debt of region 1 per capita to describe equilibrium objects.

In general, both regional governments receive transfers from the *CB*. The region  $i$  government receives a transfer of  $\phi^i T(k, b)$  from the *CB*, where  $\sum_i \phi^i = 1$ . Denote by  $T^i(k, b)$  the transfer per capita made to region  $i$  agents:  $\eta^i T^i(k, b) = \phi^i T(k, b)$ .<sup>8</sup>

We assume the *CB* is able to commit to a transfer function. We explore the implications of  $T(k, b)$  for the fiscal policy of region 1. The policy response of the *CB* noted in the introduction is embedded in the transfer function.

In our analysis, we also emphasize two monetary policy rules. The first is a fixed money supply rule in which the *CB* commits to a money growth rule of  $\sigma$ . We call this a  $\sigma$  – rule. The second is an interest rate rule in which the *CB* commits to target the real interest rate.<sup>9</sup> We call this a  $R$  – rule. For each of these two rules, we must specify the corresponding  $T(k, b)$  policy function of the *CB*.

### 3 Equilibrium Analysis

We focus on steady state equilibrium of this economy, given a stock of region 1 debt. We then characterize the dependence of the steady state equilibrium on regional debt.

Given a transfer function,  $T(k, b)$  and fiscal policy of region 1,  $b$ , a steady state equilibrium satisfies the following conditions:

- Household optimization characterized by the first order condition for a representative region  $i$  household:

$$u'(c_y^i) = Rv'(c_o^i) \tag{5}$$

$$c_y^i = \omega + g^i - \tau_y^i - s^i, \quad c_o^i = Rs^i - \tau_o^i \tag{6}$$

for  $i = 1, 2$

- budget constraint for region  $i$  government determining  $\tau_y^i$  and  $\tau_o^i$  from  $(k, b)$

$$B^i = g^i - \tau_y^i, \quad B^i r = \tau_o^i + T^i. \tag{7}$$

for  $i = 1, 2$ .

- factor market equilibrium conditions:  $\omega(k) = f(k) - kf'(k)$  and  $R(k) = f'(k)$ ,
- zero-profit condition for intermediary:  $R = (1 - \lambda)r(k) + \frac{\lambda}{\pi}$ .

Given the assumption of constant returns to scale, the number of producers is not determined in equilibrium. We assume that there is a single firm producing, hiring workers from all regions.<sup>10</sup>

<sup>8</sup>Thus the total transfer is  $T(k, b) = \sum_i \eta^i T^i(k, b)$ .

<sup>9</sup>We explore other interest pegs as well.

<sup>10</sup>An equivalent formulation would assume no capital mobility. Firms would then operate in different regions.

The integration of capital markets implies that in each period, the markets of money, capital rentals, labor and government bonds must clear. The functions  $\omega(k)$  and  $r(k)$  guarantee that the markets for capital and labor clear at the given levels of factor supply. The market for government bonds will clear as long as the government pays the market clearing rate of interest.

The analysis follows two possibilities. The first occurs when the reserve requirement is not binding. In this case, money earns the same return as other assets and, we find, that fiscal policy of one region can be completely neutral.

The second case corresponds to an economy in which the reserve requirement binds. In this case the fiscal policy of one region is not neutral and instead impacts on equilibrium wages and interest rates.

The interaction of fiscal and monetary policy will depend both on whether the reserve requirement is binding and the nature of the policy rule. As we shall see, an important issue is whether monetary policy is directly conditioned on the debt outstanding of a region or whether the response of the central bank is induced by variations in interest rates and wages.

### 3.1 Non-binding Reserve Requirements

As a useful special case, we start the analysis by studying economies in which the reserve requirement does not bind. In this case, the rates of return on money, capital and bonds must be equal in equilibrium.<sup>11</sup>

We characterize the steady state equilibria as a function of the policy chosen by the *CB* through the transfer function. We distinguish two cases: one in which the transfer function depends on  $k$  only and a second in which transfers depend directly on  $b$ .

These two cases are economically important. If the *CB* conditions its policy on  $b$ , then its response to the fiscal policy of the regional government is direct: changes in the amount of regional debt elicits a monetary response. In this case, it is not surprising that fiscal policy of one region has monetary consequences.

If, instead, the *CB* conditions policy only on  $k$ , then the link between regional fiscal actions and a monetary response will be indirect. This dependence of monetary policy on capital arises from the dependence of wages and interest rates on the capital stock. As we shall see, in this case, variations in fiscal policy which induce changes in the capital stock lead to changes in prices. Under some policy rules, a monetary response arises.

The key to this linkage between fiscal and monetary policy is the ability of regional fiscal policy to effect the aggregate capital stock. As we see in the follow propositions, when the reserve requirement is not binding, regional debt and the capital stock are unrelated. Thus the only link between fiscal and monetary policy occurs when the *CB* conditions transfers directly on  $b$ .

#### 3.1.1 Transfer Function Independent of $b$

In this case, the *CB* has a transfer function which depends only on  $k$ . Thus any feedback of monetary policy is through variations in  $k$  induced by the regional fiscal policy. From the next proposition, the steady state

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<sup>11</sup>A stationary monetary equilibrium exists in an overlapping generation model with real assets and no asset markets frictions only if the real return on assets is lower than the growth rate.

equilibrium is independent of the fiscal policy of region 1. Thus, in the steady state equilibrium, the transfers are independent of the region 1 policy.

**Proposition 1** *If  $T_b(k, b) \equiv 0$ , then the steady state equilibrium is independent of  $b$ .*

**Proof.** Using the budget constraint of the regional government in (3), the equilibrium consumption of region 1 agents is

$$c_y^1 = \omega(k) - \left(s^1 - \frac{b}{\eta^1}\right), \quad c_o^1 = \left(s^1 - \frac{b}{\eta^1}\right)R(k) + T^1(k, b) \quad (8)$$

and the consumption of region 2 agents is

$$c_y^2 = \omega(k) - s^2, \quad c_o^2 = s^2R(k) + T^2(k, b). \quad (9)$$

The first-order condition is given by (5) for  $i = 1, 2$ . Recall that  $b = \eta^1 b$ .

The only fiscal policy variable of region 1 appearing in the equilibrium conditions is the level of debt,  $\frac{b}{\eta^1}$ . Evaluating the first-order condition at the equilibrium levels of consumption, it is clear that only the difference between saving and bonds in region 1,  $(s^1 - \frac{b}{\eta^1})$ , is determined in equilibrium. This is because  $T(k, b)$  is independent of  $b$ . Thus variations in region 1 debt are matched by variations in the saving of region 1 agents. The per capita capital stock,  $k$ , and other equilibrium variables are independent of  $\frac{b}{\eta^1}$ . The transfers from the *CB* to the region 1 government are independent of  $\frac{b}{\eta^1}$ . ■

This is a standard Ricardian result. Essentially region 1 conducts fiscal policy in isolation from region 2 and from the *CB*. Variations in taxes set in youth influence the level of debt and taxes in the future. Households in region 1 fully anticipate the link between current and future taxes and adjust savings accordingly. There are no equilibrium effects of region 1 fiscal policy. That is, equilibrium interest rates and wages are independent of  $\frac{b}{\eta^1}$ .<sup>12</sup> In equilibrium the transfers of the *CB* are independent of the fiscal policy of region 1.

### 3.1.2 Transfer Function Dependent on $b$

We now consider  $T_b(k, b) \neq 0$  so the *CB* is induced to respond to region 1 fiscal policy directly. That is, even if region 1 policy has no direct effect on the capital stock, the *CB* responds directly to variations in  $b$ . Given this direct dependence of monetary policy on the fiscal policy of region 1, it is not surprising that the steady state depends on the fiscal policy stance of region 1, summarized by  $b$ .

**Proposition 2** *If  $T_b(k, b) \neq 0$ , then the steady state equilibrium depends upon  $b$ .*

**Proof.** Look again at the equilibrium consumption of region 1 agents

$$c_y^1 = \omega(k) - \left(s^1 - \frac{b}{\eta^1}\right), \quad c_o^1 = \left(s^1 - \frac{b}{\eta^1}\right)R(k) + T^1(k, b). \quad (10)$$

The level of region 1 debt now appears explicitly in the transfer function. If the steady state level of the capital stock was independent of  $b$ , then only the difference between saving and bonds in region 1,  $(s^1 - \frac{b}{\eta^1})$ , would be determined in equilibrium. But this is not the case when  $T^1(k, b)$  depends on  $b$ . Thus even if

<sup>12</sup>Thus models used to study the international coordination of fiscal policies either have non-Ricardian elements in them or study the impact of the level of government spending.

variations in region 1 debt were matched by variations in the saving of region 1 agents, the consumption level of region 1 agents would depend on  $b$ . Thus (10) would not hold. The steady state capital stock depends on  $b$ . ■

There are equilibrium effects of region 1 fiscal policy through a *CB* rule in which  $T_b(k, b) \neq 0$ . If this was the only link between regions, the remedy would be clear: the *CB* should adopt a rule in which  $T_b(k, b) \equiv 0$ . But our interest goes beyond this case and thus we now study an economy in which region 1 fiscal policy influences the capital stock more directly.

### 3.2 Binding Reserve Requirements

In this section, we study an economy in which the reserve requirement is binding. This friction in asset markets breaks the equalization of returns on money and other assets and will lead to a couple of important findings.

First, Ricardian equivalence fails: the fiscal policy choices of region 1 will have real effects on the capital stock and thus on wages and interest rates. Second, unless the *CB* adopts rule in which  $T(k, b)$  is independent of  $(k, b)$ , the fiscal policy of region 1 will induce a policy response.

We study steady state equilibria with valued fiat money given the transfer function of the *CB* and the fiscal policy of region 1. We compare steady states for different levels of debt issued by region 1.<sup>13</sup>

We will generally study the economy in the neighborhood of a particular steady state without region 1 debt and with a constant money supply. At that steady state, we will assume  $f'(k) > 1$ . If, instead we assume  $f'(k) < 1$  at the steady state, then the economy is dynamically inefficient and a role for policy already exists independent of dealing with fiscal spillovers. If  $f'(k) = 1$ , then there is no distortion due to the reserve requirement since the return on money and capital is the same. This would return us to the case studied earlier. Hence, unless stated otherwise, the analysis takes place in the neighborhood of a steady state with  $f'(k) > 1$ .

Our starting point for the analysis is to study the case in which the *CB* is not active. That is,  $T(k, b) \equiv 0$ . This case is useful as it highlights the existence of fiscal spillovers: the effects of region 1 fiscal policy on the steady state even for an inactive monetary authority.

**Proposition 3** *For  $T(k, b) \equiv 0$ , the steady state equilibrium is dependent on  $b$ .*

**Proof.** To see that region 1 fiscal policy does impact the steady state equilibrium, we assume that it does not and reach a contradiction. Suppose the region 1 government alters the level of taxes in youth and thus changes the level of debt it issues:  $\Delta \frac{b}{\eta^1} = -\Delta \tau_y^1$ . If this change in fiscal policy was neutral, then  $R$  would not change and region 1 agents would simply adjust their savings with the change in taxes. In that case, the change in aggregate saving would be given by  $\Delta S = \eta^1 \Delta \frac{b}{\eta^1}$ , where  $\eta^1$  is again the size of the region 1 population. Using  $b + k = (1 - \lambda)S$ ,  $\Delta k = (1 - \lambda)\Delta S - \Delta b$ . If the only change in aggregate saving is due to the change in region 1 debt, then  $\Delta S = \Delta b$  implies  $\Delta k = (1 - \lambda)\Delta b - \Delta b = -\lambda\Delta b \neq 0$  when  $\lambda > 0$ . This contradicts the construction of an equilibrium with neutral fiscal policy.

<sup>13</sup>We use local dynamics only to obtain signs for comparative statics. Understanding how the transfer function of the *CB* influences local dynamics is a topic for further study.

Changes  $b$  will thus effect  $k$  and hence the real interest rate and the real wage rate. The consumption profile of region 2 agents is characterized by (5) and (6). Variations in  $\omega$  and  $R$  induced by the fiscal policy of region 1 will alter the consumption levels and welfare of region 2 agents. The equilibrium allocation is not independent of  $\frac{b}{\eta^1}$ . ■

This proposition makes clear that the Ricardian result is not robust to the introduction of reserve requirements. Given the differential in return between household saving and the interest on government debt,  $r > R$ , the dependence of the steady state on region 1 fiscal policy is not surprising.

Proposition 3 assumed an inactive  $CB$ . We now ask if there exists a  $CB$  policy which eliminates the fiscal spillover highlighted by Proposition 3. As the next proposition shows, there does not exist such a monetary policy.

**Proposition 4** *There does not exist a  $T(k, b)$  transfer function such that the steady state equilibrium is independent of  $b$ .*

**Proof.** If there exists a  $T(k, b)$  rule such that the steady state was independent of region 1 fiscal policy, then both  $R$  and  $k$  would be independent of  $\frac{b}{\eta^1}$ . For this to be the case, the rate of money growth would have to be constant so that the rate of inflation was constant. If so, the FOC for a representative region 1 agent must just determine  $s^1 - \frac{b}{\eta^1}$  so the variations in  $\frac{b}{\eta^1}$  are only reflected in  $s^1$ . Substituting the region 1 budget constraint into the FOC for the region 1 representation agent, we obtain

$$u'(\omega + \frac{b}{\eta^1} - s^1) = Rv'(s^1 R - \frac{b}{\eta^1} r + T(k, b)) \quad (11)$$

With  $T(k, b) = \frac{\lambda(k+b)}{1-\lambda} \tilde{\sigma}$ , holding  $\sigma$  fixed implies that  $T_b(k, b) = \frac{\lambda}{1-\lambda} \tilde{\sigma}$ . Substituting that into the FOC above, we do not find that only  $s^1 - \frac{b}{\eta^1}$  is determined. [to be completed] ■

These two results imply that the fiscal policy of region 1 has effects on the steady state capital stock. Further, there does not exist a transfer function which would insulate region 2 agents from the fiscal policy of region 1.

With these results in mind we turn to study how the debt of region 1 effects the steady state. We do so by studying the local dynamics of the capital stock and the response of the economy to a change in the debt of region 1.

Capital dynamics are given by:

$$k_{t+1} = (1 - \lambda) \sum_i \eta^i s^i(\omega(k_t), R(k_{t+1}), \tau_y^i, \tau_o^i(k_{t+1}, b)) - b. \quad (12)$$

where  $s^i(\omega, R, \tau_y^i, \tau_o^i)$  is the savings function for a region  $i$  agent given factor prices and taxes from (5). Here  $\tau_o^i(k_{t+1}, b) = \frac{rb}{\eta^i} - T(k, b)$  as in (7).

Local dynamics are governed by:

$$\frac{dk_{t+1}}{dk_t} = \frac{(1 - \lambda) \sum_i \eta^i s^i_\omega \omega'(k_t)}{1 - (1 - \lambda) [\sum_i \eta^i s^i_R R'(k_{t+1}) + s^i_{\tau_o} \frac{\partial \tau_o^i(k, b)}{\partial k}]}. \quad (13)$$

From the region 1 budget constraint, the tax when old depends on the capital stock as well as the transfers from the *CB*.<sup>14</sup> Thus  $\frac{\partial \tau_o^i(k,b)}{\partial k} = \frac{r'b}{\eta^i} - T_k(k,b)$ .

If  $\frac{dk_{t+1}}{dk_t} \in (0,1)$  globally, then the steady state with positive capital is unique and locally stable. If the *CB* is not responsive to variations in the capital stock,  $T_k(k,b) \equiv 0$ , then this is the standard Diamond (1965) model. In that case,  $\frac{dk_{t+1}}{dk_t} > 0$  as both the numerator and the denominator are positive and  $\frac{dk_{t+1}}{dk_t} < 1$  by standard assumptions.<sup>15</sup>

In general we will allow  $T_k(k,b) \neq 0$ . If  $T_k(k,b) \geq 0$ , we will have  $\frac{dk_{t+1}}{dk_t} > 0$  since  $s_{\tau_o}^i > 0$  from consumption smoothing. This plus the Diamond conditions for stability will imply  $\frac{dk_{t+1}}{dk_t} \in (0,1)$ .

With this structure, we can then study:

$$\frac{dk}{db} = \frac{-[1 - (1 - \lambda)\eta^1 \left( s_{\tau_y} \frac{d\tau_y}{db} + s_{\tau_o} \frac{d\tau_o}{db} \right)]}{1 - (1 - \lambda)[\sum_i \eta^i s_R^i R'(k_{t+1}) + s_{\omega}^i \omega'(k_t) + s_{\tau_o}^i \frac{r'b - \phi^i T_k(k,b)}{\eta^i}]} \quad (14)$$

**Proposition 5** *If  $\frac{dk_{t+1}}{dk_t} \in (0,1)$  at the steady state and  $T_b(k,b) \geq 0$ , then  $\frac{dk}{db} < 0$ .*

**Proof.**

The denominator of (14) is positive based on local stability. From the regional government's BC,  $\frac{d\tau_y^1}{db} = \frac{-1}{\eta^1}$  and  $\frac{d\tau_o^1}{db} = \frac{r - \phi^1 T_b(k,b)}{\eta^1}$ . We use this to evaluate the numerator of (14). We also use the condition  $R s_{\tau_o} - s_{\tau_y} = 1$  from the household's optimization problem. We find:

$$\begin{aligned} -[1 - (1 - \lambda)\eta^1 \left( s_{\tau_y} \frac{d\tau_y}{db} + s_{\tau_o} \frac{d\tau_o}{db} \right)] & \\ &= -[1 + (1 - \lambda)(s_{\tau_y} + s_{\tau_o}(r - \phi^1 T_b(k,b)))] \\ &= -(1 - \lambda) \left[ \frac{\lambda}{(1 - \lambda)} (-s_{\tau_y} + \frac{s_{\tau_o}}{\sigma}) + s_{\tau_o} \phi^1 T_b(k,b) \right] \end{aligned}$$

Here we used  $R s_{\tau_o} - s_{\tau_y} = 1$ . Since  $s_{\tau_y} < 0$  and  $s_{\tau_o} > 0$ , this term, which is the numerator, will be negative if  $T_b(k,b) \geq 0$ . ■

## 4 Special Rules

These results point to the effects of region 1 debt on equilibrium outcomes and potential monetary response. As we have seen, the monetary response arises from the dependence of regional transfers on the state of the economy, represented by  $(k,b)$ .

We now turn to the two special forms of transfer functions induced by two forms of monetary policy. One specifies a money growth rate. The second is an interest rate peg.

These rules are of interest for a couple of reasons. First, they are well established rules for monetary policy. Second, they highlight a basic tension that arises in the conduct of fiscal and monetary policy within

<sup>14</sup>As the capital stock evolves, we assume that the regional government budget constraint is met by changes in taxes on agents in old tax only.

<sup>15</sup>See the discussion in Diamond (1965).



a federation. The monetary authority may adopt a non-interventionist rule, such as fixing the growth rate of the money supply, thus allowing the crowding out of capital from regional debt policy, as in Proposition 5. Or, the monetary policy may prescribe intervention to, for example, reduce or even eliminate crowding out by pegging an interest rate. But this policy may have other effects since the process of pegging an interest rates requires transfers to the regional governments. By focusing on these two cases, we are able to highlight these issues.

#### 4.1 Fiscal Spillovers under a $\sigma$ – rule

We characterize the equilibrium under a fixed money growth rule, denoted  $\sigma$  – rule. The transfers required to support a given growth rate in the money supply are given by

$$T(k, b) = \lambda S \frac{(\sigma - 1)}{\sigma} = \frac{\lambda(k + b)}{(1 - \lambda)} \tilde{\sigma} \quad (15)$$

where  $\tilde{\sigma} \equiv \frac{\sigma - 1}{\sigma}$ . Using (15), the rate of money growth is determined by the transfer policy set by the *CB* along with the demand for real money balances, which is proportional to the holdings of bonds and capital.<sup>16</sup>

With this specific policy, we again find there is crowding out:

**Proposition 6** *At a locally stable steady state, an increase in  $b$  leads to a reduction in the capital stock, an increase in the real interest rate and a reduction in the real wage.*

**Proof.** The key expression is the dynamic equation for the per capita capital stock:

$$k_{t+1} = (1 - \lambda) \sum_i \eta^i s^i (\omega(k_t), R(k_{t+1}), \tau_y^i, \tau_o^i(b, k_{t+1})) - b. \quad (16)$$

Here  $k_t$  is the per capita capital stock in period  $t$  and  $b$  is the per capita amount of region 1 debt outstanding. As the capital stock evolves, the region 1 budget constraint must be met. Thus the tax on old agents depends on the capital stock,  $\tau_o^1(b, k_{t+1})$ .

We first use (16) to study local dynamics. We then use its steady state version as a basis for comparative statics with respect to  $b$ .

Differentiating (16),

$$\frac{dk_{t+1}}{dk_t} = \frac{(1 - \lambda) \sum_i \eta^i s_\omega^i \omega'(k_t)}{1 - (1 - \lambda) \sum_i \eta^i s_R^i R'(k_{t+1}) - s_{\tau_o}^1 [bR'(k_{t+1}) - \lambda \tilde{\sigma}]} \quad (17)$$

where  $\tilde{\sigma} \equiv \frac{\sigma - 1}{\sigma}$ . We assume that both  $s_\omega > 0$  and  $s_R > 0$ . From consumption smoothing,  $s_{\tau_o} > 0$ . From the production function,  $r'(k) = f''(k) < 0$ , and  $\omega'(k) = -kf''(k) > 0$ . Hence  $\frac{dk_{t+1}}{dk_t}$  is positive as both the numerator and the denominator are positive. At a locally stable steady state,  $\frac{dk_{t+1}}{dk_t} < 1$ .

Armed with (17) and the assumption of local stability, we can look at the effects on a locally stable steady state of a change in per capita debt. To do so, we need to be clear about the fiscal policy experiment.

<sup>16</sup>Importantly, this is not the same policy as  $T(b, k) \equiv 0$ , except when  $\sigma = 1$ . Else, a fixed money growth rule creates new money and the level of transfers depends on  $(k, b)$ .

Here we follow the logic of the stability argument so that  $\tau_o$  will vary with  $k$ . Assume that the change in taxes in youth satisfies  $d\tau_y = -\frac{db}{\eta^1}$ . The tax on old age income is given by  $\tau_o^1(b, k) = [bR(k) - \frac{\lambda(k+b)}{1-\lambda}\tilde{\sigma}]$ .

With all of this, we can totally differentiate (16) and evaluate the derivative at the steady state. This yields

$$\frac{dk}{db} = \frac{-[1 + (1 - \lambda)(s_{\tau_y} - s_{\tau_o} \frac{R-\lambda}{1-\lambda})]}{1 - (1 - \lambda) \sum_i \eta^i [s_{\omega}^i \omega'(k_t) + s_R^i R'(k_{t+1})] - s_{\tau_o}^1 [bR'(k_{t+1}) - \lambda\tilde{\sigma}]} \quad (18)$$

The denominator of this expression is positive by the of local stability. The following proves that the numerator is negative. It uses the fact that  $s_{\tau_y} - s_{\tau_o}R = -1$  as  $s_{\tau_y} = \frac{u''}{-(u'' + R^2v'')}$  and  $s_{\tau_o} = \frac{-Rv''}{-(u'' + R^2v'')}$  from (5).

$$\begin{aligned} -[1 + (1 - \lambda)(s_{\tau_y} - s_{\tau_o} \frac{R-\lambda}{1-\lambda})] &= -[1 + (1 - \lambda)(-1 - s_{\tau_o} \frac{\lambda(R-1)}{1-\lambda})] \\ &= -[\lambda(1 - s_{\tau_o}(R-1))] \\ &= -[\lambda(1 + \frac{R(R-1)v''}{-(u'' + R^2v'')}] \\ &= -[\lambda \left( \frac{-(u'' + Rv'')}{-(u'' + R^2v'')} \right)] < 0 \end{aligned}$$

Hence  $\frac{dk}{db} < 0$  since the numerator is negative and the denominator is positive. ■

When the monetary authority follows a constant money growth rate policy, an increase in the debt of region 1 has the traditional crowding out effect of reducing the capital stock. Though this policy does not directly effect region 2 agents through fiscal instruments, the change in the capital stock leads to changes in real wages and interest rates. These price changes do impact on region 2 agents.

To eliminate this fiscal interaction, a monetary authority may turn to a  $R - rule$ . By stabilizing the interest rate, some of the fiscal spillovers will be eliminated. But, the monetary authority may then be induced to facilitate the financing of regional debt obligations.

## 4.2 Equilibrium under Interest Rate Targets

### 4.2.1 Real Rate Target

We consider a  $CB$  policy to peg the interest rate on deposits,  $R$ . As before, the  $CB$  is assumed to do so by making money transfers to the region 1 government. With this policy the central bank is indeed “weak”: it is forced to respond to the fiscal policy of region 1. But, remarkably, this same policy insulates the agents in region 2 from the inflation tax induced by the fiscal policy of region 1. Further, by pegging the real rate, this form of policy limits the fiscal spillover to the effect of region 1 fiscal policy on the wage rate.

If the  $CB$  follows an  $R - rule$ , the characterization of the equilibrium can be reduced to two conditions. One is the household’s FOC with equilibrium variables inserted and the other is the interest rate equation, (4). The two unknowns are  $(k, \sigma)$ .

For the household, we have  $c_y^1 = w + g^1 - \tau_y^1 - s^1$  and  $c_o^1 = s^1 R - \tau_o^1$ . We can rewrite this as  $c_o^1 = sR - \frac{b}{\eta^1} \tilde{r} + \lambda s \frac{(\sigma-1)}{\pi}$ . Using  $R = r(1-\lambda) + \frac{\lambda}{\pi}$ ,  $c_o^1 = kr + \lambda s$ .<sup>17</sup> Substituting all of this into the household's first order condition yields

$$u'(w + \frac{b}{\eta^1} - s^1) = Rv'((s^1(1-\lambda) - \frac{b}{\eta^1})r + \lambda s^1) \quad (19)$$

Given  $\frac{b}{\eta^1}$  and an interest rate, this equation determines  $k$  since  $s(1-\lambda) - \frac{b}{\eta^1} = k$  and  $r = f'(k)$ . Hence we can establish the following

**Proposition 7** *The steady state equilibrium depends on  $\frac{b}{\eta^1}$  in the  $R$ -rule case but the consumption and utility of region 2 agents depends on  $\frac{b}{\eta^1}$  only through real wages.*

**Proof.** By definition,  $S = \eta^1 s^1 + (1-\eta^1)s^2$  so  $\frac{S}{\eta^1} = s^1 + \tilde{s}^2$  where  $\tilde{s}^2 = \frac{(1-\eta^1)s^2}{\eta^1}$ . Further,  $S = \frac{k+\eta^1 \frac{b}{\eta^1}}{1-\lambda}$ . Using this, consumption of young region 1 agents can be written

$$c_y^1 = \omega + \frac{b}{\eta^1} - s^1 = \omega - \frac{b}{\eta^1} \frac{\lambda}{1-\lambda} - \frac{k}{\eta^1(1-\lambda)} + \tilde{s}^2. \quad (20)$$

Given  $k, R$ ,  $\tilde{s}^2$  is determined from (??) for region 2 agents as  $\eta^1$  is exogenous.

Consumption of old region 1 agents can be written as  $c_o^1 = s^1 R - r \frac{b}{\eta^1} + \frac{\lambda S}{\eta^1} \frac{\sigma-1}{\sigma}$  using the regional budget constraint and  $T^1 = \frac{\lambda S}{\eta^1} \frac{\sigma-1}{\sigma}$ . The real interest rate on deposits is given by  $R = r(1-\lambda) + \frac{\lambda}{\sigma}$ , where we use  $\sigma = \pi$  in the steady state. Substitute for  $(s^1, S, R)$ , to obtain

$$c_o^1 = \frac{b}{\eta^1} \frac{\lambda}{1-\lambda} + \frac{k}{\eta^1} (r + \frac{\lambda}{1-\lambda}) - R\tilde{s}^2 \quad (21)$$

Using these consumption levels in the region 1 agents first-order condition, (??), yields

$$u'(\omega - \frac{b}{\eta^1} \frac{\lambda}{1-\lambda} - \frac{k}{\eta^1(1-\lambda)} + \tilde{s}^2) = Rv'(\frac{b}{\eta^1} \frac{\lambda}{1-\lambda} + \frac{k}{\eta^1} (r + \frac{\lambda}{1-\lambda}) - R\tilde{s}^2) \quad (22)$$

The only unknowns in (22) are  $k$  and  $\sigma$  once the region 1 government sets  $\frac{b}{\eta^1}$ .<sup>18</sup> The  $k$  comes in through  $S$ ,  $\omega(k)$ ,  $r(k)$  and  $\tilde{s}^2$ . The  $\sigma$  comes in through the return  $R$ .

An equilibrium can be constructed in two steps. Given a target  $R$  of the  $CB$  and the level of region 1 debt, (22) determines  $k$ .<sup>19</sup> Once  $k$  is determined,  $\sigma$  must satisfy  $R = r(k)(1-\lambda) + \frac{\lambda}{\sigma}$  at the pegged rate.

The fact that the steady state depends on  $\frac{b}{\eta^1}$  comes immediately from (22) as consumption levels are not independent of  $\frac{b}{\eta^1}$  as long as there is a reserve requirement,  $\lambda > 0$ . In order for (22) to hold,  $k$  must vary with  $\frac{b}{\eta^1}$ .

As for region 2 agents, they are insulated from the money creation induced by the fiscal policy in region 1. Once  $R$  is set, the consumption of region 2 agents solves (??) with  $c_y^2 = \omega(k) - s^2$  and  $c_o^2 = Rs^2$ . While variations in  $\frac{b}{\eta^1}$  will induce a response by the monetary authority, the consumption levels in region 2 are not influenced by the inflation tax since  $R$  is independent of  $\frac{b}{\eta^1}$ . As before, the wage of region 2 agents in youth will be effected by  $\frac{b}{\eta^1}$ . ■

<sup>17</sup>This calculation of  $c_o^1$  assumes that the money created to peg the interest rate is transferred to the region 1 government.

<sup>18</sup>Again, given  $k, R$  is set,  $\tilde{s}^2$  is determined.

<sup>19</sup>A sufficient condition for existence with  $k > 0$  is  $\frac{b}{\eta^1}$  small enough.

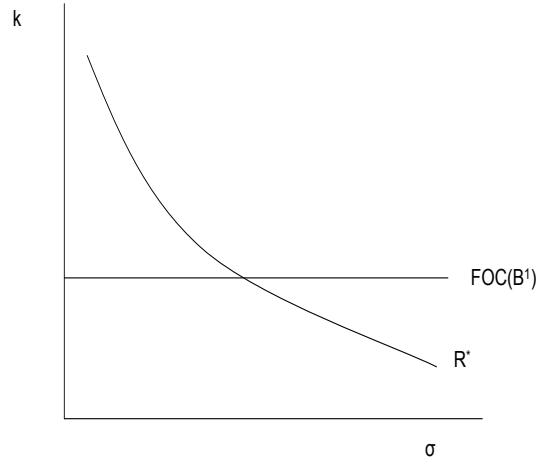


Figure 1: Equilibrium

As shown in the proof of Proposition 7, if the *CB* pegs  $R$ , then with  $\frac{b}{\eta^1}$  given, there are two equations and two unknowns,  $(k, \sigma)$  to determine. Figure ?? illustrates the construction of a steady state equilibrium. To construct this graph, we use the fact that the first-order condition of a region 1 agent is independent of  $\sigma$ : there are no income effects in equilibrium. Also, from the interest rate condition,  $k$  and  $\sigma$  must vary inversely. Thus there is a unique equilibrium characterized by the crossing of these curves.

### Equilibrium

Now consider variations in  $\frac{b}{\eta^1}$ . Proposition 8 characterizes the effects of variations in region 1 debt.

**Proposition 8** *At a locally stable steady state, an increase in  $\frac{b}{\eta^1}$  leads to a reduction in the capital stock and a reduction in the real wage.*

**Proof.** As in the proof of Proposition 6, we first study the difference equation for capital. We then look at the condition for local stability which will incorporate the effects of the per capita capital stock,  $k$ , on transfers to region 1,  $T^1$ .

The dynamic equation for the per capita capital stock:

$$k_{t+1} = (1 - \lambda) \sum_i \eta^i s^i(\omega(k_t), R(k_{t+1}), \tau_y^i, \tau_o^i(k_{t+1}, b)) - b \quad (23)$$

Here  $b = \eta^1 b$  is again the per capita amount of region 1 debt outstanding. As the capital stock evolves, the region 1 budget constraint is met by changes in  $\tau_o^1$ . Differentiating (23),

$$\frac{dk_{t+1}}{dk_t} = \frac{(1-\lambda) \sum_i \eta^i s_\omega^i \omega'(k_t)}{1 - (1-\lambda) \sum_i \eta^i s_R^i R'(k_{t+1}) + s_{\tau_o}^1 \lambda [\tilde{\sigma}(k_{t+1}) + k_{t+1} \tilde{\sigma}'(k_{t+1})]} \quad (24)$$

Using  $\tau_o^1(k, b)$ ,  $\frac{d\tau_o}{d(k_{t+1})} = -\frac{\lambda}{\eta^1(1-\lambda)} \tilde{\sigma}(k) - \frac{\lambda k}{\eta^1(1-\lambda)} \tilde{\sigma}'(k)$  captures all of the effects of the evolution of the capital stock on  $\tau_o^1$ .

Assume  $\tilde{\sigma}(k_{t+1}) + k_{t+1} \tilde{\sigma}'(k_{t+1}) > 0$ . With  $s_{\tau_o}^1 > 0$ , this implies the denominator of (24) is positive. In addition,  $s_\omega > 0$  so the numerator is positive. Thus  $\frac{dk_{t+1}}{dk_t} > 0$ .

Armed with this and the assumption of local stability, we have  $0 < \frac{dk_{t+1}}{dk_t} < 1$ . We can look at the effects on a locally stable steady state of a change in per capita debt using (23):

$$\frac{dk}{db} = \frac{-[1 - (1-\lambda)\eta^1(s_{\tau_y} \frac{d\tau_y}{d(b)} + s_{\tau_o} \frac{d\tau_o}{d(b)})]}{1 - (1-\lambda) \sum_i \eta^i [s_\omega^i \omega'(k_t) + s_R^i R'(k_{t+1})] + s_{\tau_o}^1 \lambda [\tilde{\sigma}(k_{t+1}) + k_{t+1} \tilde{\sigma}'(k_{t+1})]} \quad (25)$$

The denominator of this expression is positive by the condition of local stability. We will show that the numerator is negative.

As in the proof of Proposition 6, assume that the change in taxes in youth satisfies  $d\tau_y = -\frac{db}{\eta^1}$ . The change in the tax in old age is given by  $\frac{d\tau_o^1}{db} = \frac{R-\lambda}{\eta^1(1-\lambda)}$ . With this, (25) becomes:

$$\frac{dk}{db} = \frac{-[1 + (1-\lambda) \left( s_{\tau_y}^1 - s_{\tau_o}^1 \frac{R-\lambda}{(1-\lambda)} \right)]}{1 - (1-\lambda) \sum_i \eta^i [s_\omega^i \omega'(k_t) + s_R^i R'(k_{t+1})] + s_{\tau_o}^1 \lambda [\tilde{\sigma}(k_{t+1}) + k_{t+1} \tilde{\sigma}'(k_{t+1})]} \quad (26)$$

The numerator in this expression is exactly the same as in (18) and so is negative by the argument provided in the proof of Proposition 6. The denominator is positive by the condition of local stability. Hence  $\frac{dk}{db} < 0$ . ■

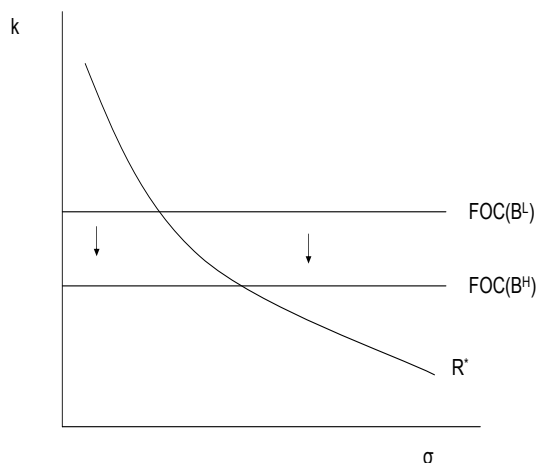
The results are illustrated in Figure ???. An increase in  $b$  from  $B^L$  to  $B^H$  will shift the first-order condition since, from (22), the level of consumption does depend on  $b$ . However, the interest rate condition does not depend on  $b$ .

As  $b$  increases, intermediaries will substitute away from capital loans to firms to the holding of debt. As usual, this crowding out leads to an increase in the marginal product of capital,  $r$ . But the *CB* responds to this by inflating in order to peg the interest rate.

Thus, as in Figure ?? we see a reduction in  $k$  and an increase in  $\sigma$  in response to the increase in  $b$ . Due to the presence of the reserve requirement,  $\lambda > 0$ , the timing of taxes does matter for real allocations: Ricardian equivalence does not hold. Instead, variations in regional debt can impact on capital accumulation.

Further, variations in regional debt can induce a response by the monetary authority. In this sense, the position of the *CB* is indeed weak relative to the  $\sigma$  – rule case.

But, do variations in  $b$  have any effect on region 2 agents? With this structure, the pegging of  $R$  by the *CB* means that region 2 agents do not bear any of the inflation tax. That is, agents in region 2 are **insulated** from the inflation induced by region 1 fiscal policy.

Figure 2: Effects of an increase in  $b$ .

In the case of a fixed money growth rule,  $b$  affected region 2 agents through the real interest rate and real wages. Under a  $R$  – rule, interest rates are independent of  $b$ . Thus the fiscal spillover is limited to real wages alone.

#### 4.2.2 Return on Capital Target

Here we peg  $r$  and thus  $k$ . so then variations in  $B$  will impact  $R$  through  $\pi$ . Given  $k$ , solve (22) for  $R$ . The LHS is falling in  $R$  and the RHS is increasing in  $R$ . so with right conditions we have a crossing. Comparative statics shows that as  $B$  increases, the LHS shifts up and the RHS shifts down so  $R$  will increase with  $B$ . Since  $r$  is fixed, the increase in  $B$  leads to lower inflation.

#### 4.2.3 Nominal Rate Target

We have seen that if the CB pegs  $R$ , then the effects of region 1 debt policy arise through variations in  $k$ . Alternatively, if the CB pegs  $r$ , then the capital stock is fixed so that variations in  $B$  come through the inflation rate.

If  $R\pi$  is pegged then so is  $r\pi$ . There is an intermediate case in which the CB pegs the nominal interest rate,  $R\pi$ . In this case, a change in region 1 debt will influence both the capital stock and the returns  $r$  and  $R$ .

To see why, consider (22). Notice that, if  $k$  does not change with  $B$ , the the LHS increases and the RHS falls. Therefore  $k$  must change. This implies that  $r$  must change and with the peg, so must  $\pi$  but so must  $R$ .

## 5 Market Segmentation

One of the critical aspects of the model studied thus far is that all agents are involved in the intermediation process. Yet, for many economies, a large fraction of agents are not involved in these markets. Instead, these agents rely largely on money as a store of value. For those agents, pegging a particular interest rate may not provide adequate insulation from the inflation tax.

To study this, we impose a very simple structure of market participation and look at its implications. In particular, we assume that only region 1 agents have access to intermediaries. Region 2 agents just hold money. This can be rationalized through the existence of costs of accessing intermediaries which differ across regions.<sup>20</sup>

The key finding here is that the insulation from the inflation tax through  $R - rule$  disappears. That is, the inflation induced by fiscal policy in region 1 is passed onto region 2 agents. In contrast, a  $\sigma - rule$  limits the spillover to the effect of region 1 fiscal policy on real wages.

### 5.1 Household Optimization

Region 1 agents have the same budget constraints and hence the same first-order conditions as earlier. For region 2 agents, their first order condition is

$$u'(w - s^2) = \tilde{\pi}v'(s^2\tilde{\pi}). \quad (27)$$

Here  $\tilde{\pi} = \frac{1}{\pi}$  is the inverse of the inflation rate. For region 2 agents,  $s^2$  is just their real money demand.

### 5.2 Intermediary

In this economy, the intermediary takes in deposits from region 1 agents and allocates them to the holding of government bonds, capital and money. As before,  $S$  is total deposits. With only region 1 agents going to the intermediary,  $S = \eta^1 s^1$ . From the reserve requirement,  $\lambda S$  is held as real money balances by the intermediary and  $(1 - \lambda)S$  is held as capital  $k$  and bonds  $b$ , i.e.  $(1 - \lambda)S = k + b$ .

In terms of returns,  $R$  is paid to deposits and  $r$  is earned on investments in bonds and in capital. The return on holding money is  $\tilde{\pi}$ . Hence  $R = (1 - \lambda)r + \lambda\tilde{\pi}$ .

<sup>20</sup>Allowing access to both forms of savings by some agents in both regions is also of interest. One could also imagine a model with endogenous market segmentation along the lines of Chatterjee and Corbae (1992). Regions may differ in terms of access to intermediaries as well as the distribution of income within a region. Both of these factors would influence the return to holding money versus deposits. It will be the regions with higher costs of intermediation and lower levels of income who would have more money holders and thus be more exposed to regional fiscal policy that induces inflation.

### 5.3 Equilibrium Consumption

Inserting the government and intermediary constraints into the budget constraints of the private agents, we can determine equilibrium consumption levels. For region 1 which has an active fiscal authority, we again find

$$c_y^1 = w - \frac{k^1}{1-\lambda} + \frac{\lambda \frac{b}{\eta^1}}{1-\lambda} \quad c_o^1 = \frac{k^1}{1-\lambda} R - \frac{b}{\eta^1} \lambda \tilde{\pi} + T^1 \quad (28)$$

where  $k^1$  is the per capita holding of capital,  $k = \eta^1 k^1$ , of region 1 agents. For region 2, consumption is given in (27).

### 5.4 Fixed Money Growth

Under a  $\sigma$  – rule, the money supply grows at a constant rate and this determines the transfer function  $T(k, b)$ . With  $\lambda > 0$ , it is not possible to satisfy (28) with a fixed value of  $k^1$  as  $b$  varies. That is,  $b$  matters for equilibrium allocations.

But, looking at the consumption levels in (27), region 2 agents are insulated from the effects of region 1 fiscal policy on interest rates. As before, the real wage is influenced by  $b$ . Thus when region 2 agents hold only real money balances, the  $\sigma$  – rule rule partially insulates them from region 1 fiscal policy.

### 5.5 $R$ – rule

In this case, as the  $CB$  works to peg the interest rate, agents in region 2 will pay the inflation tax. The interest rate is again given by  $R = (1 - \lambda)f'(k) + \lambda\tilde{\pi}$ . As before, if, for example,  $b$  increases,  $k^1$  must fall given  $R$ . This will increase  $r$  and hence  $\tilde{\pi}$  will be lower to keep  $R$  fixed so  $\pi$  will be higher.

The inflation has a welfare effect on region 2 agents since they hold money as a store of value. An increased deficit in region 1 induces a money transfer and hence a tax on region 2. This effects comes on top of the effects of  $b$  on the wage and the interest rate.

## 6 Conclusion

This paper studies the contribution of monetary policy to the interactions of regional fiscal policies. In the presence of frictions, modeled here as a reserve requirement, the fiscal policy decisions of one region will generally affect relative prices, and thus spillover to other regions.

Depending on the rule it adopts, the monetary authority can influence the extent of these spillovers. Paradoxically, some rules that appear to be ‘accommodating’ and responsive to fiscal actions of regional governments, seem to better suppress these spillovers. As we have seen, a monetary rule that pegs the interest rate may partially insulate one region from another. Surprisingly, this insulation happens despite the fact that the monetary authority is using an inflation tax to peg a real interest rate. Yet, if some agents do not participate in asset markets, this rule also weakens the monetary authority and facilitates the



exploitation of one region by another. This arises because the fiscal actions of one region can induce the monetary authority to levy an inflation tax, which affects all regions.

These results point to some more general lessons on the interaction between regional fiscal policy and the choice of monetary rules worth understanding more completely. One of our key findings is that in some cases monetary policy may serve to partially insulate one region from another. The required elements for obtaining such an insulation result are: (i) the choice of an interest rate to peg, and (ii) the extent of agents' participation in asset markets. For the insulation property to hold, it is sufficient that the monetary authority pegs the interest rate in the market in which agents of the region to be insulated are saving. However, the insulation is not complete, and some spillovers operating through changes in wage rates may persist. The insulation result fails under a monetary policy which is not characterized by (i) and (ii). Understanding more general conditions for this insulation, as well as consequent welfare implications, remain an open issue.

Our analysis has focused on steady states. As in Smith (1994), the choice of rules may also have interesting implications for local dynamics.

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