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Forced Portfolio Liquidation*

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Abstract: We study the problem of a leveraged investor that is forced to unwind a significant fraction of its portfolio in a collection of illiquid markets. It is shown that markets may become disrupted in response to a relatively small liquidity shock. As a consequence, the probability of default can be much higher than suggested by standard risk measures. We also study the impact of successful liquidation on relative asset prices. Our analysis suggests that effective risk management of leveraged financial entities should focus on the entity's potential to generate emergency cash-flows net of third-party claims for liquidity.

Keywords: Portfolio liquidation, market disruption, leverage, determinants of asset liquidity, hedge funds, structured credit.

JEL codes: G11, E58.

Résumé: Nous étudions le problème d'un investisseur au levier financier élevé, qui se voit contraint de vendre une proportion significative de son portefeuille sur des marchés illiquides. Nous montrons que les marchés peuvent s'effondrer en réponse à un choc relativement limité. Par conséquent, la probabilité de défaut peut être beaucoup plus élevée que ce que suggèrent les mesures de risque standard. Nous étudions également l'impact d'une liquidation effective sur les prix d'actifs. Notre analyse suggère que le management effectif du risque pour des entités au levier financier élevé devrait être centré sur le potentiel de mobilisation de cash-flows d'urgence, nets des créances liquides détenues par des tiers.

Mots-clefs: Liquidation de portefeuille, effondrement de marchés, levier financier, déterminants de la liquidité d'actifs, hedge funds, crédit structuré.

Codes JEL: G11, E58.

Non-technical summary: In this paper, we study the problem of a leveraged investor that is forced to unwind a significant fraction of its positions in a collection of illiquid markets. Real world examples include hedge funds, conduits used for restructuring credit, and other investments vehicles. For clarity, we assume that the investor is *fully leveraged*, i.e., the investor is assumed to have pledged all available collateral for the purpose of financing the investments undertaken. Thus, further credit would not be available for the investor.

First, we study the feasibility of successful liquidation. We find that a fully leveraged investor may fail in response to a liquidity outflow that is very small compared to the available equity base. Indeed, the investor, unable to obtain additional external funding, would have to liquidate some assets. However, when markets are not perfectly liquid, the necessary extent of liquidation may not be immediate, because the selling should depress market prices, provoke margin calls, and trigger further selling. As we show, in the worst case, there may be no subportfolio that when brought to the market would generate sufficient liquidity to satisfy creditors' request for sufficient collateralization of outstanding credit. That is, the market for illiquid assets would break down in such a scenario, rendering the valuation of such assets an ambiguous exercise. For the investor, this potential disruption of trading has the consequence that the threshold value for the loss that triggers operational default is typically much smaller than suggested by standard risk measures.

In the second part of the analysis, we study the optimal partial liquidation of a portfolio, provided it is feasible. We derive an explicit expression for the optimal liquidation strategy for the case that asset returns are jointly normally distributed, and that no single asset class must be liquidated completely. Maybe interestingly, the optimal liquidation order can be decomposed, in a first-order approximation, into a sum of a "market" portfolio and an "emergency portfolio," where the latter is constructed from the vector of collateral margins and from the variance-covariance matrix characterizing assets' return profiles. We also study the impact of a forced liquidation on asset prices. Specifically, we show that the market valuation of an individual

asset is relatively less vulnerable to a liquidity shock when creditors require a higher margin for the asset, or when it exhibits less correlation to other assets.

Overall, our findings suggest that the consideration of balance sheet data is not sufficient for managing risks of leveraged financial entities. Instead, the analysis calls for an explicit consideration of collateral pledges, market illiquidity, and potential non-availability of market prices. Supervision of leveraged funds and investment vehicles should be based on comprehensive scenario analyses focusing in particular on internal liquidity flows that can be generated by the investor over a given horizon net of third-party claims for liquidity. Effective risk management should take care that this *unencumbered cash-flow potential (UCP)* remains positive over staggered horizons with a high probability of confidence.

Résumé non-technique: Nous étudions le problème d'un investisseur au levier financier élevé qui se voit contraint de vendre une proportion significative de son portefeuille sur des marchés illiquides – comme ce peut être le cas pour les hedge funds, les conduits employés pour restructurer les instruments de crédit ou encore d'autres véhicules d'investissement. Pour clarifier la discussion, nous supposons que le levier est total, c'est-à-dire que l'investisseur a mobilisé tout son collatéral afin de financer ses investissements – un tel investisseur ne peut donc plus obtenir de crédit supplémentaire.

L'analyse se décompose en deux parties. Dans la première, nous étudions la faisabilité d'une stratégie de liquidation. Il apparaît qu'un investisseur dont le levier financier est total peut faire défaut suite à une perte, même si celle-ci correspond à une fraction seulement de ses fonds propres. En effet, afin de faire face à ses obligations, l'investisseur, incapable d'obtenir des fonds externes supplémentaires, doit se défaire d'autres titres. Cependant, lorsque les marchés ne sont pas parfaitement liquides, l'ampleur de la liquidation peut ne pas être immédiate car les cessions font baisser les prix d'actifs, provoquent des appels de marges et induisent des cessions supplémentaires. Nous montrons que dans le pire des cas, il se peut qu'aucun sous-portefeuille ne soit en mesure de générer, sur le marché, la liquidité nécessaire pour satisfaire

la collatéralisation des encours de crédits requise par les créiteurs. Dans ce scénario, le marché d'actifs illiquides s'effondre, ce qui rend la valorisation des tels actifs assez ambiguë. Pour l'investisseur, la rupture potentielle des échanges a pour conséquence de diminuer sensiblement la valeur-limite des pertes déclenchant le défaut opérationnel (à un niveau beaucoup plus bas que celui suggéré par des mesures de risque standard).

Dans la seconde partie de l'analyse, nous étudions les stratégies optimales de liquidation réalisables. Une expression explicite est dérivée au cas où les rendements d'actifs suivent une densité jointe normale et où aucune classe d'actifs ne doit être entièrement liquidée. En première approximation, l'ordre optimal de liquidation peut être décomposé en une somme de portefeuilles « de marché » et « d'urgence », ce dernier étant construit à partir des vecteurs de haircuts et de la matrice de variance-covariance qui caractérise les actifs risqués. Nous étudions également l'impact d'une liquidation forcée sur les prix d'actifs et montrons que l'évolution des prix d'un actif donné est relativement moins vulnérable à un choc de profits que lorsque les créiteurs requièrent une marge plus élevée, ou lorsqu'il exhibe une corrélation moindre avec les autres actifs.

Dans l'ensemble, les résultats suggèrent que la prise en compte de données de bilan ne suffit pas à évaluer les risques d'entités financières au degré de levier élevé. L'analyse appelle plutôt une prise en compte explicite des engagements de collatéral, de l'illiquidité de marché et de la possibilité que les prix de marché puissent cesser d'être disponibles. La supervision des fonds au levier financier élevé et des véhicules d'investissement devrait être fondée sur l'analyse de scénarios exhaustifs centrés en particulier sur les flux de liquidité internes qui peuvent être générés par l'investisseur sur un horizon donné, nets des créances liquides détenues par des tiers. Un management du risque efficace devrait s'assurer que le cash-flow disponible (*unencumbered*) potentiel reste positif sur des horizons échelonnés avec un confortable intervalle de confiance.

I. Introduction

Almost overnight, highly leveraged investment entities have found their way into the reality of the contemporary financial world. It has become evident quite recently that there may be situations in which such investment entities have to liquidate, for a reason not anticipated, a considerable fraction of their securities holdings. For instance, a hedge fund,¹ faced by an unexpected change in market conditions, may have to unwind its positions in response to calls to repay loans in lack of sufficient collateral. Is survival possible in such a situation? If so, which assets should be thrown on the market? And in which proportions? With this paper, we explore these questions, i.e., we study the conditions for successful and optimal forced liquidation of an investment portfolio in a collection of illiquid markets. We also discuss the implications of our analysis for risk management and prudential supervision of leveraged investment entities.

Our analysis has two parts. In the first part, we study the feasibility of successful selling in an illiquid market. We find here that a leveraged investor may fail in response to a loss that is only a fraction of its capital base. Indeed, the investor, once unable to obtain additional external funding, would have to start selling securities from its portfolio. The execution of the market orders has two effects. The first effect is a potential pressure on market prices. In the spirit of the interpretation of liquidity as a price for immediacy, the larger the liquidation order for a specific asset, the stronger will the pressure on the market price of that asset.² The depressed market prices will cause trading losses with the next construction of the marked-to-market balance sheet, and diminish further the investor's equity base.

The other effect of the liquidation, potentially much stronger than the first, is that the creditors, which have an eye on the collateral of their borrower, may start to call credits that are no longer sufficiently protected. In fact, typically, the investor would be unable to sell pledged collateral without

¹See, for instance, Stulz [19].

²The price impact can be mitigated, but not avoided, by stretching the liquidation over the available span of time.

the creditors' explicit consent. The investor may therefore have to explain in sufficient detail to the creditors how the liquidation strategy is going to resolve the temporary problem. However, as we show, there may be no fraction of the portfolio that, when brought to the market, would generate sufficient funds to satisfy creditors' request for sufficient collateralization of outstanding credit. Thus, there would be no offer made to the market in the first place. The market disruption occurs despite symmetric information of market participants!³

In the second part of the paper, we study the optimal partial liquidation of a portfolio, provided it is feasible. We derive an explicit expression for the optimal liquidation strategy for the case that asset returns are jointly normally distributed, and that no single asset position must be liquidated completely.⁴ Maybe interestingly, the optimal liquidation order can be decomposed, in a first-order approximation, into a sum of a "market" portfolio and an "emergency portfolio," where the latter is constructed from margin requirements and the variance-covariance matrix of the asset pool. Similarly, the impact on asset liquidity caused by an exogenous shock is composed of an ex ante liquidity term and a risk/margin term.

The investor in our model can be thought of as any sort of leveraged fund or investment vehicle. Hedge funds have already been mentioned as an example. Here, risk management is outsourced to the prime broker who is also the provider of credit to the hedge fund. Depending on the strategy chosen, hedge funds tend to focus on a trading gain that can be realized if the willingness to accept risks is sufficiently high. Leverage becomes crucial in the implementation of such a strategy because the trading margin may otherwise be too small to generate sufficient investor interest. Once the market turns against the strategy, however, there may be no way out other than reversing the investment strategy. Legal entities and special investment vehicles (SIVs) used for restructuring credit should be another example. For instance, such

³This finding suggests itself as a potential explanation of the fact, that during the liquidity crisis in August 2007, several hedge funds (including funds set up by BNPP) have been "frozen" until price levels rebounded somewhat.

⁴We provide an example with two risky assets in which one position is fully liquidated.

conduits may issue commercial paper backed by credit claims taken from the originator’s balance sheet. The originator grants credit lines for the case that commercial paper cannot be rolled over, which implies normally a good rating for the conduit. However, if there are nevertheless concerns about the quality of the assets, those credit lines will have to be used.

The following numerical example illustrates the mechanics of our theoretical framework.

Example 1. A leveraged investor, a sizable player in the community, is equipped with the following balance sheet:

Assets (\$)	Liabilities (\$)
Stocks 1,200	Equity 300
Exotics 400	Loans 1,300

The investor’s creditors, who are the only providers of loans, require that funding must be secured by collateral, where haircuts of 15 and 30 percent are applied to the stock and to the exotic, respectively. With these parameters in place, it is not difficult to verify that the investor is *fully leveraged*, i.e., the creditors would not be willing to provide additional funding for further investments.⁵ Indeed, the market value of the investor’s collateral, diminished by the respective haircut, amounts to

$$\text{Credit limit} = (100\% - 15\%) \cdot \$1,200 + (100\% - 30\%) \cdot \$400 = \$1,300.$$

In the present paper, we are interested in the general question how the investor’s balance sheet will be re-adjusted when an unexpected event occurs. For instance, the investor might suffer from an unexpected operational loss of \$50.

To identify the optimal liquidation strategy in this example, the investor needs to form expectations about the likely market impact of the liquidation.

⁵Scarcity of suitable collateral is illustrated by the fact that recently, there have been regulatory concerns in the UK that collateral has been used twice by hedge funds. Our conclusions obviously apply also to investors that are not fully leveraged.

We assume here that initially, the market price of the stock and the exotic has been \$10 each (so that the investor has 120 and 40 securities, respectively, of each class in her portfolio). The expected appreciation of the stock and of the exotic investment would be approximately +\$5 and +\$11 in the long run. The uncertainty in the returns is captured by standard deviations of \$1 for the stock and \$2 for the exotic investment, the correlation coefficient being zero in this example. Assuming that the market's parameter of absolute risk aversion is 0.1, and ignoring potential indivisibilities, our results imply that it would be optimal to sell 34 stocks and 22 exotic investments, with a current market value of about \$560, which is more than tenfold the loss that needs to be covered!

Why so much? The direct price impact of the liquidation is not the main driver. Indeed, as a consequence of the liquidation, market prices would fall to \$9.66 for the stock and to \$9.12 for the exotic.⁶ These variations are clearly covered by the haircuts applied by the investor's creditor. The cash flow resulting from the settlement of the market order would be

$$\text{Cash flow} = 34 \times \$9.66 + 22 \times \$9.12 \approx \$527.$$

Thus, the liquidation value generated is only \$33 lower than the market value of the investor's assets before the liquidation. There must be another effect driving the excessive liquidation. The point to note here is that the combined impact of lowered prices and smaller number of securities held reduces the total value of the investor's collateral basis. It is not difficult to check that the investor's balance sheet after the liquidation is given by

Assets (\$)	Liabilities (\$)
Stocks 833	Equity 174
Exotics 164	Loans 823

In fact, after the liquidation, the credit limit of the fully leveraged investor

⁶Please note that figures are conveniently rounded in this example.

reduces to merely

$$\begin{aligned} & \text{Credit limit after loss} \\ &= (100\% - 15\%) \cdot \$833 + (100\% - 30\%) \cdot \$165 = \$823. \end{aligned}$$

Thus, the difference amount of \$477 to the earlier \$1300 will be requested immediately and *in cash* by the creditors. The investor's net cash inflow is therefore

$$\text{Net cash flow} = \$527 - \$477 = \$50,$$

which is just enough to pay the bill that caused the problem in the first place. Moreover, the investor's equity position has shrunk from \$300 to \$174, in response to an unexpected loss of only \$50!⁷

Our formal discussion draws essential elements from several existing contributions. Possibly most closely related is the paper by Chowdry and Nanda [5], who study trading under margin requirements in a single risky asset. The present paper can be considered as an extension of their model to the multi-asset case, with some modification to the price formation that will be discussed in Section II. In their paper, Chowdry and Nanda have argued that margin requirements can lead to instability when investors trade repeatedly, which provides a rationale for the use of price limits. In our set-up, which is simpler in this regard because there is only one investor, the problem is better-behaved, i.e., multiplicity cannot obtain even when no price limits are imposed by the market regulator.⁸

Duffie and Ziegler [8] study a related problem for an investor subject to capital regulation. With the help of a Monte Carlo approach, they evaluate various intertemporal liquidation strategies in markets with stochastic bid-ask spreads. It is shown that selling liquid assets first minimizes transaction costs, whereas selling illiquid assets keeps the probability of default low.

⁷Banks are not typically highly leveraged due to the fact that credit claims are not accepted as collateral in the interbank market. However, to the extent that illiquid collateral is accepted by central banks, commercial banks may be able to become fully leveraged in the future.

⁸An informal description of the pyramiding-depyramiding process of investment can be found in Garbade [9]. Similar mechanics are discussed by Kiyotaki and Moore [14], and by Cifuentes, Ferrucci, and Shin [6].

Another closely related contribution is by Brunnermeier and Pedersen [3], who discuss multiplier effects resulting from the endogeneity of margin requirements, and the role of funding of the market making sector for system stability. In contrast to the present study, the market making sector is risk-neutral and capital constrained in Brunnermeier and Pedersen’s model (thus, the modeling assumptions concerning the risk attitudes of investors/customers and the market making sector are just exchanged). Also the market mechanism is different. While Brunnermeier and Pedersen consider a Walrasian price formation, we consider quantity-setting à la Cournot.⁹

The rest of the paper is structured as follows. Section II presents the basic set-up and introduces our equilibrium notion. In Section III, we discuss the optimal liquidation order in response to an unexpected loss. Section IV offers some extensions. Section V concludes. All proofs can be found in the Appendix.

II. The basic model

We consider an economy with $K + 1$ assets, with $K \geq 1$. One asset, asset 0 (cash), is riskless, serves as a numeraire, and is the only source of terminal utility for the agents in the economy. The other K assets, assets $k = 1, \dots, K$, are risky assets. We think here mainly of financial assets, but in principle physical assets that allow alternative uses should also be consistent with our interpretation.

Risky assets will be illiquid in our framework because market participants request a premium for temporarily accepting additional risks.¹⁰ As an example

⁹There are further related contributions, including the following. Acharya and Pedersen [1] offer a model of asset pricing, in which liquidity risk is reflected by an autoregressive process of illiquidity costs. Vayanos [20] offers a dynamic model of an asset market with transaction costs. Vayanos and Vila [21] and Huang [13] study the relationship between liquidity and asset prices. Brunnermeier and Pedersen [2] discuss how illiquid markets allow a stronger market participant to take advantage of a temporary weakness of another market participant. The fact that constraints on financial wealth can have contagious consequences is captured in models by Kyle and Xiong [15] and Xiong [22].

¹⁰Indeed, as Grossman and Miller [11] have shown, market participants should generally

for an illiquid asset, consider collateralized debt obligations (CDOs). Indeed, being tailor-made constructions in which a portfolio of credits is collected on the asset side of a special vehicle, CDOs tend to be difficult to get rid of in times of distress. Risk management is non-trivial. The liability side is typically structured in three main parts. A senior tranche is debt offered to the most risk-averse clientele, the mezzanine tranche to investors that seek potentially more risky high-yield bonds, and the equity tranche is offered even more risk-seeking entities (such as hedge funds).¹¹

There are three dates $t = 0, 1, 2$. Initial endowments are held at date 0, assets are traded at date 1, and payoffs collected at date 2. Returns from risky assets at date 2 are jointly normally distributed with expected value

$$v = (v^1, \dots, v^K)',$$

and invertible variance-covariance matrix Ω . Here and throughout the paper, we denote by X' the transpose of a vector or matrix X .¹²

The economy is inhabited by a single risk-neutral institutional investor and a continuum (of mass one) of risk-averse traders. The assumption of trader homogeneity is made for convenience only. There seems to be no principle problem with dropping this assumption.

have to compensate the market maker for offsetting demand and supply shocks over time. A number of papers have worked successfully with the assumption of risk-aversion on the part of counterparties that stand ready to take both sides of the market. Greenwald and Stein [10] extend the base model and show that uncertainty about the number of (risk-averse) buyers leads to an inefficient allocation of risk because the buyers shy away from the resulting uncertainty about the market price. Diamond and Verrechia [7] show that expected illiquidity can lead to a depreciation of an asset. I.e., when an investor may be forced to liquidate a position in the future, this may lower the value of the position today. Empirical evidence for a risk-averse market-making sector is provided by Campbell, Grossman, and Wang [4], and by Pastor and Stambaugh [16]. Schnabel and Shin's [17] model is also consistent with this assumption.

¹¹While balance sheets of vehicles that transport CDOs are also fully leveraged, with significant embedded leverage, there is a difference to actively managed investment strategies in that CDOs face no risk management constraint. The investor in the senior debt, for instance, would be in a role different from that of a prime broker in a hedge fund, because she cannot ask for additional collateral should the quality of the underlying portfolio deteriorate.

¹²As we disallow negative positions, the invertibility of the variance-covariance matrix is indeed a mild restriction.

The *investor* is equipped with an exogenous amount $e_0 > 0$ of equity (or capital), has outstanding credit (collateralized loans) of $D_0 \geq 0$, holds cash $C_0 \geq 0$, and a portfolio

$$x_0 = (x_0^1, \dots, x_0^K)'$$

of risky assets, subject to the balance sheet equation

$$C_0 + p_0^1 x_0^1 + \dots + p_0^K x_0^K = e_0 + D_0.$$

Here, initial market prices of the risky assets

$$p_0 = (p_0^1, \dots, p_0^K)'$$

at date 0 are treated as exogenous, with $p_0^k > 0$ for $k = 1, \dots, K$. Consistent with our interpretation of assets as positions of a balance sheet, we assume $x_t^k \geq 0$ for $k = 1, \dots, K$ and for $t = 0, 1, 2$, i.e., no short-selling.

To model margin requirements, we assume that the investor's total credit must not exceed the market value of her portfolio, where asset-specific haircuts are applied to individual positions by the providers of credit. For instance, the investor may be financed through revolving asset-backed commercial paper (ABCP). Formally, the risk management constraint is satisfied when

$$D_0 \leq C_0 + (1 - h^1)(p_0^1 x_0^1) + \dots + (1 - h^K)(p_0^K x_0^K), \quad (1)$$

where $h^k > 0$ denotes the risk weight (haircut) applicable to risky asset $k = 1, \dots, K$. Rewriting (1) in the form of a minimum capital requirement yields

$$h^1(p_0^1 x_0^1) + \dots + h^K(p_0^K x_0^K) \leq e_0, \quad (2)$$

where now, the parameter h^k has the interpretation of an asset-specific capital risk-weight. Haircuts are exogenous to our model. Apparently, the cash component disappears in (2) because the haircut for cash pledged as collateral is, as we assume, just zero. In the sequel, for simplicity, $C_0 = C_1 = 0$, so that loans are net of cash.¹³

¹³We assume that the risk management constraint (2) is the only financial covenant that creditors impose on the borrowing entity, and that equity cure rights are not granted to the sponsor.

Between dates 0 and 1, the investor receives an unanticipated bill $\pi_1 \leq 0$ that will have an impact on the investor's balance sheet at date 1. We think here of an exogenous change in the equity position that is not caused by trading but instead, for instance, by operational gains or losses (e.g., a realized legal risk), sudden equity withdrawals, and the like.¹⁴ To deal with the liquidity shock, the investor submits at date 1 a liquidation order

$$\Delta_1 = (\Delta_1^1, \dots, \Delta_1^K)$$

to the market. The sign convention is that $\Delta_1^k < 0$ refers to an order to sell, while $\Delta_1^k > 0$ corresponds to an order to buy. With the execution of the liquidation order, the investor's new portfolio (at date 1) is given by the sum $x_1 = x_0 + \Delta_1$. The vector of transaction prices at date 1 will be some $p_1 = p_1(\Delta_1)$. We will describe the price formation in our asset market below. The change in prices affects the value to the existing portfolio x_0 which, via the balance sheet equation at date 1 increases or decreases the investor's equity position to

$$e_1 = \underbrace{e_0}_{\text{equity at date 0}} + \underbrace{\pi_1}_{\text{liquidity shock}} + \underbrace{(p_1 - p_0)'x_0}_{\text{trading loss}}.$$

The limit order Δ_1 will be said to be *feasible* (i.e., avoids credit calls from the creditors that would necessitate further liquidations) if the risk management constraint

$$h^1(p_1^1 x_1^1) + \dots + h^K(p_1^K x_1^K) \leq e_1$$

at date 1 is satisfied for $p_1 = p_1(\Delta_1)$.

At date 2, all assets create their returns, and the investor ends up with terminal wealth (equity at date 2) of

$$e_2 = e_1 + (v - p_1)'x_1.$$

It is assumed that the investor is risk neutral and maximizes e_2 .

¹⁴Alternative triggers for forced liquidations that can be discussed within the current framework include the down-grading of assets, changed return expectations, and an reduction in the market's risk appetite.

The representative *trader* is endowed, at date 0, with a portfolio

$$y_0 = (y_0^1, \dots, y_0^K)'$$

We assume $y_0 \neq 0$, and $y_0^k \geq 0$ for $k = 1, \dots, K$. At date 1, the market receives the market order Δ_1 from the investor and offers the transaction price $p_1(\Delta_1)$.

The microstructure of the asset market has been chosen to reflect the tension between the large investor and the many small traders. Specifically, we envisage a Cournot style of price formation, with the special form of a monopoly in the case of a single institutional investor. That is to say, the investor chooses quantities, and can commit to those quantities, so that supply becomes perfectly inelastic to changes in the price. Then the market price is formed in a Walrasian fashion between the inelastic supply of the investor and the aggregate demand formed by individual traders in the market. Thus, the investor chooses (a vector of) quantities, while the representative trader is a price taker in this market model. This type of price formation circumvents the celebrated schizophrenia problem identified by Hellwig [12]. The problem would be that a large investor who is aware of her market impact cannot properly be considered as a price taker. Note also that in extension to the standard Cournot model, the individual investor could alternatively commit to buying, rather than selling a certain quantity of a specific asset.¹⁵

At date 2, the representative trader has accumulated a terminal wealth of

$$\Pi_2 = \tilde{v}'y_0 + (\tilde{v} - p_1)'\Delta_1,$$

where \tilde{v} is the assets' return vector realized at date 2. It is assumed that the representative trader has a utility function with a constant coefficient of absolute risk aversion $\gamma > 0$.¹⁶

¹⁵An alternative set-up in which a market order implies a price impact under symmetric information assumes a zero-profit condition for market makers. However, this approach would not be useful for two or more risky assets because the price vector would not be well defined as a consequence of potential cross-subsidization across trading desks.

¹⁶It will be noted that we are considering a liquidation under symmetrically shared

We will now formally define what we mean by an equilibrium in our model. Fix a negative profit shock $\pi_1 < 0$, i.e., a loss.

Definition 1. *A pair $(p_1^*(.), \Delta_1^*)$ will be called a liquidation equilibrium if (a) the investor submits a market order Δ_1^* so as to maximize her expected terminal wealth, given $p_1^*(.)$, and (b) the representative trader's inverse demand function is given by $p_1^*(\Delta_1)$.*

The liquidity shock is assumed to occur at date 1, and will be denoted by π_1 . There are at least two interpretations for the shock. One interpretation considers the shock as an unexpected operational loss for the investor, e.g., due to the realization of uncovered legal risks. In another interpretation, where the investor merely manages equity provided by other investors, there could be an unexpected withdrawal of funds, for instance, as suggested by Shleifer and Vishny [18].

The optimal liquidation strategy is derived as follows. With endogenous market prices at date 1 of

$$p_1 = (p_1^1, \dots, p_1^K)',$$

profit accounting based on the balance sheet equation

$$\max\{\pi_1; 0\} + p_1^1 x_0^1 + \dots + p_1^K x_0^K = \max\{-\pi_1; 0\} + e_1 + D_0$$

which holds at date 1 (after trading but before settlement) delivers the following expression for equity at date 1:

$$e_1 = e_0 + \pi_1 + (p_1^1 - p_0^1)x_0^1 + \dots + (p_1^K - p_0^K)x_0^K$$

Thus, capital at date 1 is the sum of the capital endowment at date 0, the profit shock, and the wealth effect caused by a change in the market prices between dates 0 and 1.

information, especially concerning the portfolio of the distressed investor. This assumption has been made for tractability and may not always be satisfied in reality. However, in the case of company shares, significant fractions held in a specific company, with the threshold value depending on the legislation applying to the company, imply a publication of the investment. Positions may also be difficult to hide completely because securities will typically be held through a custodian, who may seek to exploit its informational advantage in case of a liquidation even if not allowed to do so.

The change in the level of equity leads either to the possibility of further investment (when $\pi_1 > 0$) or to a forced liquidation (when $\pi_1 < 0$). Also at date 1, the investor's debt must be collateralized, which, as mentioned earlier, is tantamount to the capital requirement

$$h^1(p_1^1 x_1^1) + \dots + h^K(p_1^K x_1^K) \leq e_1, \quad (3)$$

where $x_1^k = x_0^k + \Delta_1^k$ for $k = 1, \dots, K$ is the investor's position in asset k at date 1. Thus, at date 1, the investor chooses a vector of market orders

$$\Delta_1 = (\Delta_1^1, \dots, \Delta_1^K)'$$

so as to maximize expected equity at date 2

$$\begin{aligned} e_2 &= e_1 + (v^1 - p_1^1)x_1^1 + \dots + (v^K - p_1^K)x_1^K \\ &= e_0 + \pi_1 + (v^1 - p_0^1)x_0^1 + \dots + (v^K - p_0^K)x_0^K \\ &\quad + (v^1 - p_1^1)\Delta_1^1 + \dots + (v^K - p_1^K)\Delta_1^K, \end{aligned}$$

subject to the risk-management restriction (3) at date 1 and to the inverse market supply $p_1 = p_1(\Delta_1)$.

The following notation turns out to be very useful

$$H = \begin{pmatrix} h^1 & 0 & \dots & 0 \\ 0 & h^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h^K \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}.$$

The investor minimizes the lost appreciation gain caused by the liquidation, with a correction term for the savings in interest. Formally, the investor's problem then looks as follows:

$$\begin{aligned} &\max_{\Delta_1} (v - p_1(\Delta_1))' \Delta_1 \\ &\text{s.t.} \\ &\text{(i)} \quad p_1(\Delta_1)' H(x_0 + \Delta_1) \leq e_0 + \pi_1 + (p_1(\Delta_1) - p_0)' x_0 \\ &\text{(ii)} \quad x_0 + \Delta_1 \geq 0 \end{aligned}$$

Constraint (i) is the risk-management constraint at date 1. Restriction (ii) is a consequence of the earlier assumption of not allowing negative positions.

While endowments and prices at date 0 are exogenous, they must conform to certain restrictions to be economically sensible.

Definition 2. *We will say that a tuple (x_0, y_0, p_0, v, e_0) is an ex-ante equilibrium if the corresponding liquidation equilibrium for $\pi_1 \equiv 0$ satisfies $p_1^*(\Delta_1^*) = p_0$, and $\Delta_1^* = 0$.*

Assumption 1. (Ex-ante equilibrium) *(x_0, y_0, p_0, v, e_0) is an ex-ante equilibrium.*

Let μ_0 denote the Lagrangian multiplier for constraint (i).

Lemma 1. *Fix γ, H, e_0 , and Ω . Then in any ex-ante equilibrium with $p_0^k < v^k$ for some k , and with a continuous inverse supply function $p_1(\cdot)$, the investor's risk management constraint (2) is binding, i.e., $\mu_0 > 0$. Moreover, the K -dimensional bordered manifold of ex-ante equilibria can be parameterized, for instance, by the vector (x_0, p_0, μ_0) .*

In principle, if initial equity e_0 were sufficiently large, then the investor would prefer a portfolio that leaves the risk management constraint unbinding. However, this cannot be the case when asset prices still reflect a risk premium. Intuitively, a fully invested portfolio must be subject to the creditors' risk management constraint, because otherwise, there would be an incentive to invest further.

The manifold of ex-ante equilibria is bordered because asset positions x_0 and y_0 must be nonnegative, and expected appreciation $v - p_0$ must also be nonnegative. Note also that the Lagrangian multiplier μ_0 has a straightforward interpretation as the shadow cost of missing capital, or the (marginal) return on equity.

Assumption 1 is necessary to create the reference point for marked-to-market margin requirements of "normal" market conditions from which deviations

due to unexpected shocks can be analyzed. In principle, it would be feasible to have the economy start at an earlier stage in which the investor enters the market with a given equity position, and begins investing. However, as intuitively, the investor's continuously growing position should ultimately result in an ex-ante equilibrium as a steady state, we avoid the complication of analyzing the build-up phase in more detail.

Lemma 2. *Under Assumption 1, the market price p_1^* at date 1 in any liquidation equilibrium is given by*

$$p_1^*(\Delta_1) = p_0 + \gamma\Omega\Delta_1. \quad (4)$$

Moreover, the investor's choice-set is compact, and the liquidation problem allows at most one solution.

Thus, as in Grossman and Miller [11], the market price reflects the limited risk-taking capacity of the market, which implies a liquidity premium for a transaction executed without too much delay. In particular, if there is selling in the short term, then market prices will typically be driven down relative to initial values.

III. Default

If hedge funds were to be regulated, how should that be accomplished? In this section, we argue that risk management based on balance sheet data may be largely ineffective for leveraged investors. To make our point, we introduce the following two concepts.

Definition 3. *The investor defaults legally when $e_0 + \pi_1 < 0$. The investor defaults operationally when the investor's problem does not allow a solution.*

In practice, if a hedge fund defaults operationally, then there should be renegotiations between the prime broker and the hedge fund how to proceed. One course of action is that the prime broker considers the credits as a default, and liquidates the collateral, potentially at a loss. Other possible courses

of action suggested by the model involve an equity cure, or a temporary lowering of haircuts by the prime broker.

To study the conditions under which operational and legal default obtains, we reformulate the investor's problem using Lemma 2. Replacing $p_1(\Delta_1)$ by the explicit formula for the price at date 1 yields the equivalent problem

$$\begin{aligned} & \max_{\Delta_1} (v - p_0 - \gamma\Omega\Delta_1)' \Delta_1 \\ & \text{s.t.} \\ & \text{(i')} (p_0 + \gamma\Omega\Delta_1)' H(x_0 + \Delta_1) \leq e_0 + \pi_1 + (\gamma\Omega\Delta_1)' x_0 \quad (5) \\ & \text{(ii)} x_0 + \Delta_1 \geq 0 \quad (6) \end{aligned}$$

Also from Lemma 2, we know that the investor's problem allows at most one solution. However, it should be noted that, like under the realities of a significant loss, there may be no market order that allows the investor to satisfy her creditors' concerns for sufficient collateral. Because the effect appears to belong to the folklore on illiquid markets, we state our formal result as an observation.

Theorem 1 (Folk Theorem for fully leveraged investors). *A legal default is always also operational. However, with illiquid markets, legal default may occur even when the investor's equity is large enough in size to cover an unexpected loss or an unexpected withdrawal of capital.*

In the case of an operational default, the illiquid market breaks down completely, i.e., there is no offered quantity for which the investor would generate sufficient funds to avert the imminent default. Obviously, the above statement has implications for capital and liquidity regulation. Specifically, the folk theorem shows that capital regulation based on the notion of equity is an inappropriate notion for investment vehicles that are fully leveraged. For instance, when risks measured by value-at-risk figures are compared to the capital of the fully leveraged investor, then it would be a misperception to believe that the total equity would be a buffer to the loss that may occur in, say, 99 out of 100 cases. The missing piece is collateral risk that needs to be

added to the value-at-risk figure to make capital regulation work in the case of fully leveraged investment vehicles. However, quantifying the collateral risk presupposes an assessment of the liquidity of individual assets.

Example 2. As an illustration, consider an investor with capital $e_0 = 200$ that has taken up loans $D_0 = 600$ to finance a total of 100 assets with marked-to-market valuation $p_0 = 8$. The asset has an expected appreciation of $v - p_0 = 10.5$, and a standard deviation of $\sigma = 1$. The market's risk aversion is $\gamma = 0.02$ in this example. The investor's prime broker requesting a haircut of $h = 25\%$, the investor can be seen to be fully leveraged, with a marginal return on equity of $\mu_0 = 6$. One can show now (this would follow from Theorem 2 below) that for a loss $\pi_1 < -12.5$, the investor's problem does not allow a solution. Thus, the unencumbered cash-flow potential (UCP) will salvage the investor from bankruptcy only for losses up to the dimension of about 7% of the investor's equity base.¹⁷

The consequence of this example for regulation is that, in order to assess the probability of operational default, the value-at-risk percentile for a given duration (e.g. 10 days) should be compared to the UCP over the same time horizon. In fact, it is easy to see that the UCP will always be strictly smaller than the investor's equity base, unless the investor holds only cash. We will see in Section V that the UCP will be even smaller when there are other investors in the economy with a binding risk management constraint.

IV. Optimal liquidation

In this section, we consider the scenario where default can be circumvented without external assistance, and look at the optimal liquidation strategy. Thus, it will be assumed from now onwards that the investor's loss is not

¹⁷This example should capture well the problems that could arise from so-called covenant-lite structures in leveraged buy-outs, in which the borrower is constrained only by leverage ratio defined as loans over ebitda. According to a press statement by Clifford Chance lawyer Guido Hoffmann, this type of financing structure has been used in Europe the first time in March 2007 (VNU). Moreover, the take-over of Alliance Boots by KKR has been reported to have a similar structure.

too large, so that bankruptcy is avoided and a liquidation strategy is well defined.

In fact, to keep things simple, we will assume even that the shock is small enough so that the investor keeps all assets in the portfolio. In principle, when the loss is large enough, the optimal liquidation may lead to the full liquidation of individual positions. For instance, in Example 1, a loss of \$81 would induce the investor to sell all of her position in asset 2. Following our interpretation of assets as nonnegative positions in the investor's balance sheet, we would find the constraint $x_1^2 \geq 0$ binding for losses larger than \$81. Thus, for losses exceeding a certain threshold level, the liquidation would involve full liquidation of some assets, and partial liquidation of others. To avoid the complications caused by the additional constraints, we assume in the following the short-selling constraint (ii). Thus, we impose the following assumption.

Assumption 2. *The investor's problem at date 1 allows a solution in which the short-selling constraint is not binding.*

As discussed above, this assumption says that the investor's problem is solvable without total liquidation of one or several asset classes. We need one more piece of notation. The *margin-risk matrix*

$$H^\Omega = \frac{1}{2}(H + \Omega^{-1}H\Omega)$$

will play a central role in the formalism. It combines information about margin requirements and the risk structure of assets. In Example 5 below, we derive a more explicit expression in the case of two risky assets.

The necessary first-order condition for the optimal liquidation order Δ_1 reads

$$v - p_0 - 2\gamma\Omega\Delta_1 - \mu_1\{H(p_0 + \gamma\Omega\Delta_1) + \gamma\Omega H(x_0 + \Delta_1) - \gamma\Omega x_0\} = 0, \quad (7)$$

where $\mu_1 = \mu_1(e_1)$ is the Lagrange multiplier that depends on the investor's equity e_1 at date 1. Re-arranging (7) yields the following result.

Lemma 3. *Under Assumptions 1 and 2, the optimal liquidation order at date 1 in response to an unexpected loss or capital drain (not too large) is given by*

$$\Delta_1^* = -\frac{\mu_1 - \mu_0}{2\mu_0}(I + \mu_1 H^\Omega)^{-1}y_0, \quad (8)$$

where μ_1 is the Lagrangian multiplier associated with the investor's risk management constraint. The resulting market price p_1^* at date 1 is given by

$$p_0 - p_1^* = \frac{\mu_1 - \mu_0}{2\mu_0}(I + \mu_1 H^\Omega)^{-1}(v - p_0). \quad (9)$$

That the matrix H^Ω is the sum captures the fact that a marginal increase in the market order Δ_1 has two consequences. On the one hand, prices for the various assets adapt, which leads under marked-to-marked accounting to a modified capital requirement. On the other hand, the investor's portfolio composition is affected, which changes also capital requirements.

Using the matrix Taylor series expansion, we obtain as first-order approximation

$$\begin{aligned} \Delta_1^* &\sim \{I + \mu_1 H^\Omega\}^{-1}y_0 \\ &\approx \underbrace{y_0}_{\text{Market portfolio}} - \frac{\mu_1}{2} \underbrace{H^\Omega(v - p_0)}_{\text{"Emergency portfolio"}}. \end{aligned}$$

Thus, the optimal liquidation path is close to a convex combination of the market portfolio and a portfolio that reflects the risk management dimension. For a relatively low opportunity cost of capital, the market portfolio is weighted stronger, whereas, for a relatively high opportunity cost of capital, the risk management dimension plays the more dominant role. Intuitively, there is a trade-off for the investor between minimizing the use of own capital and allowing the market to sustain a well-diversified portfolio. In times of stress, we would expect that μ_1 is large, so that the trade-off is essentially one-sided on the risk-management constraint.

Given the generality of Lemma 3, it is instructive to look at a number of special cases.

Example 3. Consider $H = \eta I$, where $0 < \eta < 1$ is a common haircut, applicable to all risky assets. In this special case, it is feasible to calculate the optimal liquidation order explicitly.

Theorem 2. *Impose Assumptions 1 and 2. Then, for $H = \eta I$, the optimal liquidation order in response to a profit shock $\pi_1 < 0$ is given by $\Delta_1^* = \alpha y_0$, where*

$$\alpha = \frac{1}{2\eta\mu_0} \left(\sqrt{1 + \frac{4\eta\mu_0^2}{\gamma y_0' \Omega y_0} \pi_1} - 1 \right)$$

is the common percentage change of liquidity of all K assets in response to a loss of $\pi_1 < 0$.

Thus, for the case of common haircuts, the liquidation vector points into the same direction as the market portfolio. As the next example shows, when risk weights differ across assets, however, this need not longer be the case. This should be intuitive because the investor's capital resources gain to an equal extent from the liquidation of any of the risky asset. The example will thereby underline the crucial role of margin requirements for the optimal liquidation strategy. But margin requirements are not the sole determinant. As we will see below Lemma 3 shows that for two assets with identical risk characteristics and identical margin requirements, the more illiquid one is more heavily sold in times of investor distress.

When there is an unexpected loss ($\pi_1 < 0$), then the liquidation function is concave in π_1 , due to the wealth effect that further increases the absolute size of the liquidation vector as the loss increases.

Lemma 3 shows that for a certain range of realizations of the profit shock π_1 , the investor could in principle avoid the temporary illiquidity by taking up additional loans. This will always be the case if the profit shock is smaller than the investor's equity. However, as credits are granted only against suitable collateral, there will be a need for liquidation. Prices are driven down by the market orders, which causes a loss of wealth for the investor. The consequence is that the investor becomes bankrupt. Thus, it appears

that the risk management constraint is driving the investor into bankruptcy here.

Example 4. For another special case, assume now that asset returns are uncorrelated, so that Ω is a diagonal matrix as H . Then $H^\Omega = H$, and the liquidation vector is again collinear to the market portfolio. The price movement of asset k reads

$$p_0^k - p_1^k \sim \frac{v^k - p_0^k}{1 + \mu_1 h^k}. \quad (10)$$

Equation (10) suggests that the price effect of the profit shock has two main determinants. One is the liquidity of the asset under normal market conditions, as captured by the difference between market price and fundamental value at date 0. The second factor is the haircut.

Theorem 3. *For two assets with identical liquidity at date 0, the one with the higher margin requirement exhibits a weaker reaction to a profitability shock.*

The reason for this reaction is that a higher margin leads to weaker investment under normal market conditions, because the haircut determines the cost of capital that must be employed to invest in the asset. If two assets have identical liquidity under normal market conditions, but one has a higher haircut than the other, then the asset with the higher haircut must be relatively more attractive for the investor, for instance, because of a better risk characteristics. Given this background, it is intuitive that the asset with the higher haircut reacts less severely under market stress compared to an asset with the lower haircut.

Example 5. Consider the case of $K = 2$ assets, with

$$\Omega = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \text{ and } H = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix},$$

where $\sigma_1 > 0, \sigma_2 > 0$ are the standard deviations of the two risky assets and $-1 < \rho < 1$ their correlation coefficient. This case is useful, as the potentially

somewhat intransparent term H^Ω attains a relatively simple form

$$H^\Omega = H + \frac{h_1 - h_2}{2} \frac{\rho}{1 - \rho^2} \begin{pmatrix} \rho & \frac{\sigma_2}{\sigma_1} \\ -\frac{\sigma_1}{\sigma_2} & -\rho \end{pmatrix}.$$

The margin-risk matrix can be seen to be different from H only if two conditions are satisfied at the same time: Haircuts must differ and asset returns must be correlated. For instance, when the haircut for asset 1 is higher than that for asset 2, and assets are positively correlated, then the margin-risk matrix will have upper-row entries increasing in the haircut differential. In the optimal liquidation order, an increasing correlation would imply a lower weight given to asset 1.

Indeed, Lemma 3 implies that the optimal liquidation order Δ^* is collinear to

$$\begin{aligned} & (I + \mu_1 H^\Omega)^{-1} y_0 \\ &= \left\{ \begin{pmatrix} 1 + \mu_1 h_2 & 0 \\ 0 & 1 + \mu_1 h_1 \end{pmatrix} + \mu_1 \frac{h_1 - h_2}{2} \frac{\rho}{1 - \rho^2} \begin{pmatrix} -\rho & -\frac{\sigma_2}{\sigma_1} \\ \frac{\sigma_1}{\sigma_2} & \rho \end{pmatrix} \right\} y_0. \end{aligned}$$

In the sequel, we will consider, for asset k , the ratio

$$\lambda^k = \frac{p_0^k - p_1^{k,*}}{v^k - p_0^k}.$$

This ratio measures the percentage change in asset k 's liquidity caused by an external shock, say, in the investor's capital. An explicit determination of λ^k is feasible only in special cases, e.g., when the risk weights for the individual assets are identical (see Example 3). However, it is often feasible to determine the relative size of λ^{k_1} compared to λ^{k_2} for some $k_2 \neq k_1$. An asset k_1 will be said to be *more elastic* than asset k_2 if $\lambda^{k_1} > \lambda^{k_2}$.

Theorem 4. *For two assets with correlation $\rho > 0$, assume that $h_1 > h_2$. Keeping μ_1 and y_0 fixed, an increase in ρ implies that the price impact on asset 2 will be relatively stronger than the price impact on asset 1.*

Thus, with increasing positive correlation, assets with lower margin requirements tend to be more elastic in response to selling pressure than assets with higher margin requirements. The reason for this result is again the ex-ante effect discussed above, i.e., the fact that among assets with similar characteristics concerning liquidity, return and risk, assets with lower margin requirements should be found more often in the portfolios of leveraged investors. Our result shows that selling pressure will lead to more liquidation of assets with low margin requirements.¹⁸

VII. Conclusion

During the month just past, August 2007, the financial sector has gone through a dramatic re-appraisal of the risks contained in structured credit. As a consequence of these developments, several hedge funds and special investment vehicles stumbled into severe problems, in particular because established financial channels through collateralized credit and asset-backed commercial paper (ABCP) turned out to be unsustainable under market stress. Financing could not be prolonged because creditors became concerned about market valuations of illiquid assets such as collateralized debt and loan obligations (CDOs and CLOs).

With the help of our theoretical framework, we have argued that valuations for illiquid assets may be inherently unavailable in the moment when the management of leveraged entities turns to the market to sell those assets. Specifically, we showed that the distressed investor in search of financing by temporary disinvestment may “get stuck” because, even though the long-term valuation of its assets significantly exceeds the amount needed in cash, there may be no subportfolio which, when brought to the market, would create sufficient revenue to guarantee survival.¹⁹

¹⁸Theorem 4 could be related to the fact that in the liquidity crisis caused by problems with subprime loans mid August 2007, managers of adversely affected investment funds claimed to have faced completely unanticipated relative price movements.

¹⁹This market disruption, if inefficient, may be a rationale for an initiative launched by a third party such as a lender of last resort.

In the case that a partial liquidation through the market place can indeed save the investor from bankruptcy, the composition of the portfolio that should be sold will determine the liquidity of individual assets. We have shown that higher margins make assets more liquid in a liquidation event caused by an unexpected loss or capital drain. Moreover, high correlation to other assets is detrimental to the liquidity of the individual asset.

Overall, our findings suggest that the consideration of balance sheet data is not sufficient for managing risks of leveraged funds and investment vehicles. Our analysis suggests that the probabilities obtained by standard methods may be much too low for leveraged investors. In fact, as we showed, marked-to-market accounting and value-at-risk figures may become meaningless for such legal entities, suggesting that the alleged “confidence crisis” might even have a legitimate motivation.

As a remedy to this problem, our findings call for an explicit consideration of collateral pledges, market impact, and the potential non-availability of market prices. Supervision of leveraged financial entities should be based on comprehensive scenario analyses focusing in particular on internal liquidity flows that can be generated by the investor over a given horizon net of third-party claims for liquidity. Indeed, effective risk management should take care that this *unencumbered cash-flow potential (UCP)* remains positive over staggered horizons with a high probability of confidence.

Appendix: Proofs.

Proof of Lemma 1. To provoke a contradiction, assume that $\mu_0 = 0$. But then $\Delta_1 = 0$ cannot be an ex-ante equilibrium. To see why, assume that the investor increases Δ_1^k , for some k such that $p_0^k < v$, by a small $\varepsilon > 0$. Then, because inverse supply is continuous, $p_1^k < v$, and $(v - p_1(\Delta_1))' \Delta_1 > 0$. Thus, $\Delta_1 = 0$ cannot be optimal. This proves that $\mu_0 > 0$. Now, when $\mu_0 > 0$, we have

$$e_0 = p_0' H x_0.$$

We evaluate the first-order condition (7) at $\Delta_1 = 0$ and obtain

$$\frac{1}{\mu_0\gamma}\Omega^{-1}(v - p_0) = \frac{1}{\gamma}\Omega^{-1}Hp_0 - (I - H)x_0. \quad (11)$$

Solving for v yields

$$v = (I + \mu_0H)p_0 - \gamma\Omega(I - H)x_0.$$

Using Lemma A.1 in (11) delivers

$$y_0 = \mu_0\left(\frac{1}{\gamma}\Omega^{-1}Hp_0 - (I - H)x_0\right). \quad (12)$$

This shows that e_0 , v , and y_0 can be determined uniquely from x_0 , p_0 , and μ_0 . \square

Proof of Lemma 2. Expected value and variance of the representative trader's terminal wealth are given by

$$\begin{aligned} E[\Pi_2] &= v'y_0 - (v - p_1)'\Delta_1, \\ V[\Pi_2] &= (y_0 - \Delta_1)'\Omega(y_0 - \Delta_1). \end{aligned}$$

Hence, the certainty equivalent of the risky terminal wealth is

$$E[\Pi_2] - \frac{\gamma}{2}V[\Pi_2] = v'y_0 - (v - p_1)'\Delta_1 - \frac{\gamma}{2}(y_0 - \Delta_1)'\Omega(y_0 - \Delta_1).$$

The K -dimensional first-order condition with respect to Δ_1 reads

$$v - p_1 - \gamma\Omega(y_0 - \Delta_1) = 0.$$

Re-arranging yields

$$p_1(\Delta_1) = v - \gamma\Omega(y_0 - \Delta_1).$$

Assumption 1 implies that $p_1(0) = p_0$. Hence, the first assertion. The objective function has a Hessian -2Ω . As the variance-covariance matrix Ω , which has been assumed to be invertible, is positive definite, the objective function is strictly concave. To secure uniqueness of the solution, the risk management constraint should define a convex set. This is guaranteed by Lemma A.3. \square

Proof of Theorem 1. To generate a contradiction, assume that $e_0 + \pi_1 < 0$ and that the risk management constraint (i) is satisfied. But then clearly, because we start from an ex-ante equilibrium, we have also $e_1 < 0$, which is tantamount to

$$D_1 > C_1 + p_1^1 x_1^1 + \dots + p_1^K x_1^K.$$

As prices and positions are nonnegative, clearly also

$$D_1 > C_1 + p_1^1(1 - h^1)x_1^1 + \dots + p_1^K(1 - h^K)x_1^K,$$

which again is tantamount to

$$e_1 < p_1^1 h^1 x_1^1 + \dots + p_1^K h^K x_1^K$$

in contradiction to the risk management constraint. Constructing an example where an operational default for $K = 1$ is not a legal default is now straightforward. See Example 2 (following the statement of Theorem 1). \square

Lemma A.1. *Under Assumption 1,*

$$y_0 = \frac{1}{\gamma} \Omega^{-1}(v - p_0).$$

Proof. Immediate from the proof of Theorem 1. \square

Lemma A.2. *Under Assumption 1, the investor's risk management constraint (5) is equivalent to*

$$\gamma \Delta_1' \Omega (H \Delta_1 + \frac{y_0}{\mu_0}) \leq \pi_1. \quad (13)$$

Proof. The risk management constraint

$$(p_0 + \gamma \Omega \Delta_1)' H (x_0 + \Delta_1) \leq e_0 + \pi_1 + (\gamma \Omega \Delta_1)' x_0$$

can be multiplied out into

$$p_0' H x_0 + p_0' H \Delta_1 + \gamma \Delta_1' \Omega H x_0 + \gamma \Delta_1' \Omega H \Delta_1 \leq e_0 + \pi_1 + \gamma \Delta_1' \Omega x_0.$$

Using the risk management constraint at date 0, we find that

$$\gamma\Delta'_1\Omega H\Delta_1 \leq \pi_1 + \gamma\Delta'_1\Omega((I - H)x_0 - \frac{1}{\gamma}\Omega^{-1}Hp_0).$$

Using (12) delivers (13).

Lemma A.3. *The matrix ΩH is positive definite. In particular, the investor's problem allows at most one solution.*

Proof of Lemma A.3. By the Sylvester criterion, ΩH is positive definite if and only if all of the leading principal minors are positive. As H is a diagonal matrix, the k -th leading principal minor of the matrix ΩH , for $k = 1, \dots, K$, is just the product of the k -th leading principal minor of Ω and the k -th leading principal minor of H , respectively. Since both Ω and H are positive definite by assumption, this proves the assertion. \square

Proof of Lemma 3. Sorting the terms in the first-order condition (7) yields

$$\gamma(2\Omega + \mu_1(H\Omega + \Omega H))\Delta_1 = v - p_0 - \mu_1\{Hp_0 + \gamma\Omega(H - I)x_0\}.$$

Multiplying with $1/(2\gamma)\Omega^{-1}$ from the left, we obtain

$$(I + \frac{\mu_1}{2}(\Omega^{-1}H\Omega + H))\Delta_1 = \frac{1}{2\gamma}\Omega^{-1}(v - p_0) - \frac{\mu_1}{2}\{\frac{1}{\gamma}\Omega^{-1}Hp_0 + (H - I)x_0\}. \quad (14)$$

By Lemma A.1 and equation (12), in any ex ante equilibrium,

$$y_0 = \frac{1}{\gamma}\Omega^{-1}(v - p_0) = \mu_0(\frac{1}{\gamma}\Omega^{-1}Hp_0 + (H - I)x_0). \quad (15)$$

Using this in (14) yields

$$(I + \frac{\mu_1}{2}(\Omega^{-1}H\Omega + H))\Delta_1 = \frac{\mu_0 - \mu_1}{2\mu_0}y_0.$$

This proves (8). Multiplying (8) by $\gamma\Omega$ from the left and replacing y_0 using (15) yields the assertion. \square

Proof of Theorem 2. By Lemma A.2,

$$\gamma\Delta'_1\Omega(H\Delta_1 + \frac{y_0}{\mu_0}) = \pi_1.$$

Using $H = \eta I$, we find

$$\gamma \Delta_1' \Omega \left(\eta \Delta_1 + \frac{y_0}{\mu_0} \right) = \pi_1. \quad (16)$$

From Lemma 3,

$$\Delta_1^* = \alpha y_0, \quad (17)$$

where

$$\alpha = \frac{\mu_0 - \mu_1}{2\mu_0(1 + \eta\mu_1)}. \quad (18)$$

Plugging (17) into (16) and solving for α yields

$$\alpha = \frac{1}{2\eta\mu_0} \left(\sqrt{1 + \frac{4\eta^2\mu_0^2}{\gamma y_0' \Omega y_0} \pi_1} - 1 \right).$$

Hence the assertion. \square

Proof of Theorem 3. The assertion follows immediately from equation (10). \square

Proof of Theorem 4. See the text before the Theorem. \square

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