Hot and Cold Seasons in the Housing Market*

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Abstract

Every year housing markets in the UK and the US experience systematic above-trend increases in both prices and transactions during the second and third quarters (the "hot season") and below-trend during the fourth and first quarters (the "cold season"). Seasonality in house prices poses a challenge to standard models for durable goods. To account for seasonality, this paper develops a matching model that emphasizes the role of match-specific quality between the buyer and the house and the presence of thick-market-effects in housing markets. It shows that a small, deterministic driver of seasonality can be amplified and revealed as deterministic seasonality in transactions and prices, quantitatively mimicking the seasonal fluctuations in transactions and prices observed in the UK and the US. The model can be applied to the study of lower-frequency fluctuations in housing markets.

Key words: housing market, thick-market effects, search-and-matching, seasonality, house price fluctuations.

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1 Introduction

A rich empirical and theoretical literature has been motivated by dramatic boom-to-bust episodes in regional and national housing markets.[1] Booms are typically defined as times when prices rise and there is intense trading activity, whereas busts are times when prices and trading activity fall below trend.

While the boom-to-bust episodes motivating the extant work are relatively infrequent and of unpredictable timing, this paper shows that in several housing markets, booms and busts are just as frequent and predictable as the seasons. In particular, in most regions of the UK and the US, every year a housing boom of considerable magnitude takes place in the second and third quarters of the calendar year (the "hot season"), followed by a bust in the fourth and first quarters (the "cold season").[2] The predictable nature of house price fluctuations (and transactions) is furthermore confirmed by estate agents, who in conversations with the authors observed that during winter months there is less activity and prices are lower. Perhaps more compelling, publishers of house price indexes go to great lengths to produce seasonally adjusted versions of their indexes, usually the versions that are published in the media. As stated by publishers:

"Houses prices are seasonal with prices varying during the course of the year irrespective of the underlying trend in price movements. For example, prices tend to be higher in the spring and summer months when more people are looking to buy." (From Halifax Price Index Methodology.)[3]

The first contribution of this paper is to systematically document the existence and quantitative importance of these seasonal booms and busts.[4] For the UK as a whole, we find that the difference

[1] See for example Stein (1995), Muellbauer and Murphy (1997), Genesove and Mayer (2001), Krainer (2001), Brunnermeier and Julliard (2008), and the contributions cited therein.

[2] Since we use "constant quality" house price indexes, changes in prices are not driven by changes in the characteristics of the houses transacted.

[3] A similar statement from Nationwide House Price Index Methodology: *"House prices are higher at certain times of the year irrespective of the overall trend. This tends to be in spring and summer..."* and *"...we seasonally adjust our prices because the time of year has some influence. Winter months tend to see weaker price rises and spring/summer see higher increases all other things being equal."*

[4] Studies on housing markets have typically glossed over the issue of seasonality. There are a few exceptions, albeit they have been confined to only one aspect of seasonality (e.g., either quantities or prices) or to a relatively small geographical area. In particular, Goodman (1993) documents pronounced seasonality in *moving patterns* in the US, Case and Shiller (1989) find seasonality in prices in Chicago and—to a lesser extent—in Dallas, and Hosios and Pesando (1991) find seasonality in prices in the City of Toronto; the latter conclude "that individuals who are willing to purchase against the seasonal will, on average, do considerably better."

in annualized growth rates between hot and cold seasons is above 8 percent for nominal prices (6 percent for real prices) and 108 percent for transactions. For the US as a whole, the corresponding differences are above 3 percent for nominal (and real) prices and 148 percent for transactions, though there is considerable variation within the country (particularly for prices).[5]

The predictability and size of seasonal fluctuations in house prices pose a challenge to standard models of durable-good markets. In those models, anticipated changes in prices cannot be large: If prices are expected to be much higher in June than in December, then buyers will shift their purchases to the end of the year, narrowing down the seasonal price differential. More concretely, in standard models, house prices reflect the present discounted value of a presumably long stream of flow values. Thus, seasonality in rental flows or service costs has to be implausibly large to generate seasonality in house prices.[6] A possible explanation for why standard no-arbitrage conditions fail is of course that transaction costs are very high and hence investors do not benefit from arbitrage. Still, the question remains as to why presumably informed buyers do not try to buy in the low-price season. Furthermore, it is not clear why we observe a systematic seasonal pattern. (The lack of scope for seasonal arbitrage does not necessarily imply that most transactions should be carried out in one season, nor does it imply that prices and transactions should be correlated.) To offer answers to these questions, we develop a search-and-matching model for the housing market. The model more realistically captures the process of buying and selling houses and it can more generally shed new light on the mechanisms governing housing market transactions and prices.

The model builds on two elements of the housing market that we think are important for understanding seasonality in house prices. The first element is a search friction. There are potentially two search frictions that buyers and sellers face: one is locating a vacant house (or a potential buyer), and the other is determining whether the house (once found) is suitable for the buyer (that is, a sufficiently good match). The first friction is, in our view, less relevant in the housing market context because advertising by newspapers, real estate agencies, property websites, etc. can give sufficient

[5] Our focus on these two countries is largely driven by the reliability and quality of the data.

[6] Using the standard no-arbitrage condition, we show that seasonality in housing markets does not seem to be driven by seasonal differences in rentals or service costs (see discussion in Appendix 7.2.) Similarly, it does not appear to be driven by liquidity related to overall income. Income is typically high in the last quarter, a period in which house prices and the volume of transactions tend to fall below trend. Beaulieu and Miron (1992) and Beaulieu, Miron, and MacKie-Mason (1992) show that in most countries, including the UK and the US, income peaks in the fourth quarter of the calendar year. There is also a seasonal peak in output in the second quarter, and seasonal recessions in the first and third quarters. House price seasonality thus is not in line with income seasonality: Prices are above trend in the second and third quarters.

information to buyers in order to locate houses that are ex ante in the acceptance set. But houses have many idiosyncratic features that can be valued differently by different buyers: two individuals visiting the same house may attach different values to it. We model this match-specific quality as a stochastic variable which is fully revealed after the buyer inspects the house. The second element of the model is the notion that in a market with more houses for sale, a buyer is more likely to find a better match, what we refer to as "thick-market effect."[7] Specifically, we assume the distribution of match-specific quality in a market with more houses first-order stochastically dominates the distribution in a market with fewer houses.

The model hence starts from the premise that the utility potential buyers derive from a house is fully captured by the match-specific quality between the buyer and the house. This match-specific quality is more likely to be higher in a market with more buyers and houses due to the thick-market-effect. In a thick market (or hot season), better matches are more likely to be formed, and this leads to a higher volume of transactions. Because better matches are formed on average, prices will also be higher, provided that the sellers have some bargaining power. This mechanism leads to a higher number of transactions and prices in a hot season when there are more buyers and sellers.

In the housing market this pattern is repetitive and systematic. The same half year is a hot season and the same half year is a cold season. The higher match-specific quality in the hot season can account for why potential buyers are willing to buy in the hot (high-price) season. But if our amplification mechanism is to explain seasonality, it has to answer two additional questions: (i) Why do some sellers are willing to sell in the cold (low-price) season? In other words, why is there no complete "time agglomeration," whereby markets shut down completely in a cold season? and (ii) Why is the pattern systematic, that is, why do hot and cold markets predictably alternate with the seasons?

To answer these two questions, we embedded the above mechanism into a seasonal model and study how a deterministic driver of seasonality can be amplified and revealed as deterministic seasonality in transactions and prices in the housing market due to the thick-market-effects on the match-specific quality. By focusing on a periodic steady state, we are studying a deterministic cycle and agents are fully aware that they are in such a cycle with both transactions and prices fluctuating between high and low levels across the two seasons.

[7] The labor literature distinguishes the thick-market effects due to faster arrival of offers and those due to the quality of the match. Our focus is entirely on the quality effect. See for example Diamond (1981), Petrongolo and Pissarides (2006) and Gautier and Teulings (2008).

Our answer to the first question is related to the presence of search frictions in the presence of match-specific quality. Any seller in the cold season can decide whether to sell now or wait until the hot season. If a buyer then arrives and a match can be made, the seller has to decide whether to keep searching or to sell at the possibly lower price. If he waits until the hot season, he can get on average a higher price, provided he finds a buyer with a good match. There is, however, a probability that he will not find such buyer to make a transaction; this uncertainty created by the search friction is not present in a standard asset-pricing model, in which agents can always transact at market prices.

Our answer to the second question, why the hot and cold seasons are systematic, is related to our assumption about the desire to move and the seasonal variations in it. We claim that the arrival of the exogenous process by which households want to move (the "propensity to move") has a seasonal component. In the summer months it is higher because, for example, of the school calendar: Families may prefer to move in the summer, before sending their children to new schools, or from other factors, such as weather. These differences alone, however, cannot explain the full extent of seasonality we document (in the data, seasonality in houses for sale is much lower than seasonality in the volume of transactions).[8] Most of the explanatory power of the model is due to the thick-market-effects on match-quality. We show that a slightly higher ex-ante probability of moving in a given season (which increases the number of buyers and sellers) can trigger thick-market effects that make it appealing to all other existing buyers and sellers to transact in that season. This amplification mechanism can thus create substantial seasonality in the volume of transactions; the extent of seasonality in prices, in turn, increases with the bargaining power of sellers. Intuitively better matches in the hot season imply higher surpluses to be shared between buyers and sellers; to the extent that sellers have some bargaining power, this leads to higher prices in the hot season. The calibrated model can quantitatively account for most of the seasonal fluctuations in transactions and prices in the UK and the US.

The contribution of the paper can be summarized as follows. First, it systematically documents seasonal booms and busts in housing markets. Second, it develops a search-and-matching model that can quantitatively account for the seasonal patterns of prices and transactions observed in the UK and the US. Understanding seasonality in house prices can serve as a first step to understanding

[8] For example, parents of school-age children account for only a small fraction of total movers. (See Goodman, 1993.) And although weather may make house search more convenient in the summer, it is unlikely that this convenience is worth so much money to the typical buyer. Furthermore, we find that cities with benign weather throughout the year, such as Los Angeles and San Diego, also display strong seasonality.

how housing markets work and what the main mechanisms governing housing market fluctuations are, and, as such, it can help to put restrictions on the class of models needed to characterize housing markets. In other words, seasonals in house prices, what economists and publishers typically ignore or correct for, can contain important information to guide the development and selection of appropriate models for housing markets.

The search-and-matching framework has been applied before to the study of housing markets (see for example Wheaton (1990), Williams (1995), Krainer (2001), and Albrecht et al. (2007)). Recent work on housing market fluctuations, such as Novy-Marx (2009), Diaz and Jerez (2009) and Piazzesi and Schneider (2009), adopt an aggregate matching function (as in Pissarides (2000) chapter 1) and focus on the role of market tightness (the ratio of the number of buyers to the number of sellers) in determining the probability of transaction. We distinguish the probability of making a contact and the probability that the house turns out to be a good match. The contact probability is always 1 in our model, but the match quality drawn is a random variable. In this sense, our setup is closest to Jovanovic (1979), which also emphasizes the stochastic nature of the match-specific quality for the labour market, and Krainer (2001) for the housing market. In contrast to previous models that focus on market tightness, transactions and prices in our set-up are governed by the distribution of match-specific quality.

Krainer (2001) and Novy-Marx (2009) also refer to "hot and cold" markets; however, the nature as well as the meaning of hot and cold markets is different from our paper. The key idea in Novy-Marx (2009) is that, if for any reason the ratio of buyers to sellers (or market tightness) unexpectedly increases, houses can sell more quickly, decreasing the stock of sellers in the market. This in turn increases the relative number of buyers to sellers even more, amplifying the initial shock. As a result, the bargaining position of sellers improves, leading to higher prices. Thus, all the amplification effect operates through market tightness. In our model instead, market tightness plays no role; indeed, it is constant across seasons. When an agent receives a shock that forces her to move, she becomes a potential buyer and a potential seller simultaneously and overall tightness does not change. The amplification mechanism in our model comes instead from the quality of the matches. In the summer, there are both more buyers and more sellers; the availability of a bigger stock of vacant houses improves the overall efficiency of the market, as buyers are more likely to find a better match. Put differently, our explanation relies on market thickness (the numbers of buyers and sellers) and its effect on the quality of matches, whereas Novy-Marx's hinges on tightness. This difference leads to crucially different predictions for the correlation between prices and transactions.

In Novy-Marx (2009), the number of transactions is not necessarily higher when prices are high. His model generates a positive correlation between prices and tightness, but not necessarily a positive correlation between prices and the volume of transactions, which is one of the salient features of housing markets (Stein 1995). Specifically, in Novy-Marx (2009), a large increase in the number of sellers and buyers that does not alter tightness would not alter prices at all, even if it substantially increases the number of transactions. Our model instead naturally generates a positive correlation between prices and transactions. As Wheaton (1990) has pointed out, moving homes most of the time means both selling a house and buying another and hence a model in which tightness plays a subdued role is appealing in this context. A hot market in our model is a market with high prices, more buyers and sellers and an unambiguously larger number of transactions.

"Hot-and-cold markets" in our paper are also different from those in Krainer (2001), who studies the response of housing markets to an aggregate shock that affects the fundamental value of houses; his model cannot generate quantitatively meaningful fluctuations in prices unless the aggregate shock is very persistent. A deterministic cycle in his model is equivalent to setting the persistence parameter to zero, in which case his model predicts virtually no fluctuation in prices. Our setup is different from Krainer (2001) in that it brings in thick-market effects, which, due to their amplification, are able to generate quantitatively large fluctuations in transactions and prices.[9]

The paper is organized as follows. Section 2 presents the motivating empirical evidence. Section 3 introduces the model. Section 4 presents the qualitative results and a quantitative analysis of the model, confronting it with the empirical evidence. Section 5 discusses the efficiency properties of the model and studies the robustness of the results to alternative modelling assumptions. In particular, this Section discusses optimal policy responses in the presence of booms and busts. Section 6 presents concluding remarks. Analytical derivations and proofs are collected in the Appendix.

2 Hot and Cold Seasons in the Data

In this Section we study seasonality in housing markets in the US and the UK at different levels of geographic disaggregation. We focus on the US and the UK because of the availability of constant-quality house price series in both countries.[10] As said, publishers of house price indexes produce

[9] Also, and unlike Krainer (2001), we model the endogenous evolution of the number of vacancies and buyers over time.

[10] The quality adjustment mitigates concerns with compositional changes in the types of houses transacted across seasons. Results for other countries are available from the authors. (Though we find qualitatively similar seasonal

both seasonally adjusted (SA) and non-seasonally adjusted (NSA) series. For transactions, the US National Association of Realtors (NAR) also produces NSA and SA series. In Appendix 7.1 we report the seasonal component implied by their adjustment. In our analysis, we use exclusively the (raw) NSA series to compute the extent of seasonality.

2.1 Data

UK

In the UK two main sources provide quality-adjusted NSA house price indexes: One is the Department of Communities and Local Government (DCLG) and the other is Halifax, one of the country's largest mortgage lenders.[11] Both sources report regional price indexes on a quarterly basis for the 12 standard planning regions of the UK, as well as for the UK as a whole. The indexes calculated are 'standardized' and represent the price of a typically transacted house. The standardization is based on hedonic regressions that control for a number of characteristics, including location, type of property (house, sub-classified according to whether it is detached, semi-detached or terraced, bungalow, flat), age of the property, tenure (freehold, leasehold, feudal), number of rooms (habitable rooms, bedrooms, living-rooms, bathrooms), number of separate toilets, central heating (none, full, partial), number of garages and garage spaces, garden, land area, road charge liability, etc. These controls adjust for the possibility of seasonal changes in the composition of the set of properties (for example, shifts in the location or sizes of properties transacted).

The two sources differ in three respects. First, DCLG collects information from a sample of all mortgage lenders in the country, while the Halifax index uses all the data from Halifax mortgages only, which account for an average of 25 percent of the market (re-mortgages and further advances are excluded in both cases). Second, DCLG reports the price at the time of completion of the

patterns in other countries, we are less confident about the comparability of the data, as typically they are not quality adjusted.)

[11]Other price publishers, like Nationwide Building Society, report quality adjusted data but they are already SA (the NSA data are not publicly available). Nationwide Building Society, however, reports in its methodology description that June is generally the strongest month for house prices and January is the weakest; this justifies the SA they perform in the published series. In a somewhat puzzling paper, Rosenthal (2006) argues that seasonality in Nationwide Building Society data is elusive; we could not, however, gain access to the NSA data to assess which of the two conflicting assessments (Nationwide Building Society's or Rosenthal's) was correct. We should perhaps also mention that Rosenthal (2006) also reaches very different conclusions from Muellbauer and Murphy (1997) with regards to lower-frequency movements. Finally, the Land Registry reports data on average prices, without adjusting for quality.

transaction, while Halifax reports the price at the time of approval of the mortgage. Completion takes on average three to four weeks following the initial agreement, but some agreed transactions do not reach completion. Finally, the DCLG index goes back to 1968 for certain regions, while Halifax starts in 1983.

To compute real price indexes, we later deflate the house price indexes using the NSA retail price index (RPI) provided by the UK Office for National Statistics.

As an indicator of the number of transactions, we use the number of mortgages advanced for home purchases; the data are collected by the Council of Mortgage Lenders (CML) and are also disaggregated by region.

US

The main source of NSA house price indexes for the US is OFHEO; we focus on the purchase-only index, which starts in 1991:01. This is a repeat-sale index calculated for the whole of the US and also disaggregated by Census regions and states. The repeat-sale index, introduced by Case and Shiller (1987), measures average price changes in repeat sales of the same properties; as such, the index is designed to control for the characteristics of the homes sold.[12] We also use the Standard and Poor's (S&P) Case-Shiller house price series for cities.

To compute real price indexes, we use the NSA consumer price index (CPI) provided by the US Bureau of Labor Statistics.[13]

Data on the number of transactions come from the National Association of Realtors (NAR), and correspond to the number of sales of existing single-family homes. The data are disaggregated into the four major Census regions.

2.2 Extent of Seasonality

We focus our study on deterministic seasonality, which is easier to understand (and to predict) for buyers and sellers (unlikely to be all econometricians), and hence most puzzling from a theoretical point of view. In the UK and the US, prices and transactions in both the second and third quarters

[12] The Case-Shiller approach significantly limits the extent to which changes in the composition of the sample of houses transacted can influence the price index. Specifically, using information on the values of the same physical units at two points in time controls for differences in housing attributes across properties in the sample.

[13] As it turns out, there is little seasonality in the US CPI index, a finding first documented by Barsky and Miron (1989), and hence the seasonal patterns in nominal and real housing prices coincide. The CPI is reported at monthly frequency. We take the last month of the quarter to deflate nominal prices.

are above trend, while in both fourth and first quarters they are below trend. For ease of exposition, we group data into two broadly defined seasons—second and third quarter, or "hot season," and fourth and first quarter, or "cold season." (We use interchangeably the terms hot season and summer term to refer to the second and third quarters and cold season and winter term to refer to the first and fourth quarters.)

In the next set of Figures, we depict in dark (red) bars the average (annualized) price increase from winter to summer, $\ln\left(\frac{P_S}{P_W}\right)^2$, where P_S is the price index at the end of the hot season and P_W is the price at the end of the cold season. Correspondingly, we depict in light (blue) bars the average (annualized) price increase from summer to winter $\ln\left(\frac{P_{W'}}{P_S}\right)^2$, where $P_{W'}$ is the price index at the end of the cold season of the following year. We plot similar Figures for transactions.

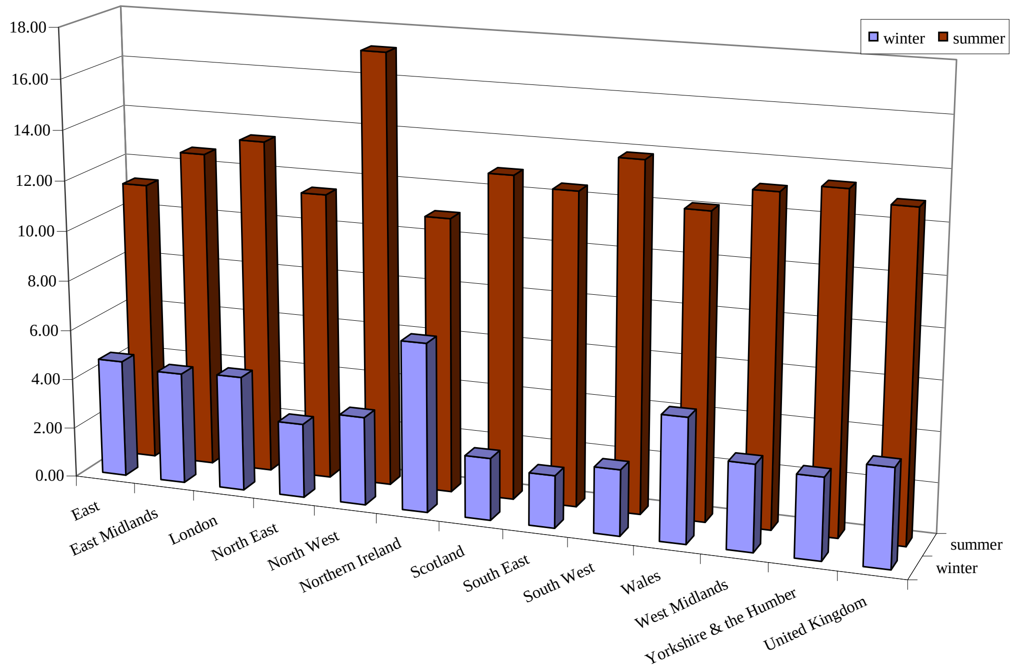
The extent of seasonality for each geographical unit can then be measured as the difference between the two bars. This measure nets out lower-frequency fluctuations affecting both seasons. (In the model we later present, we use a similar metric to gauge the extent of seasonality.)

2.2.1 Housing Market Seasonality in the UK

Nominal and Real House Prices Figure 1 reports the average annualized percent price increases in the summer term and the winter term from 1983 through to 2007 using the regional price indexes provided by DCLG. During the period analyzed, the average nominal price increases in the winter term were below 5 percent in all regions except for Northern Ireland. In the summer term, the average growth rates were above 12 percent in all regions, except for Northern Ireland, East Anglia, and the North East. As shown in the graph, the differences in growth rates across the two broad seasons are generally very large and economically significant, with an average of 9 percent for all regions. (For some regions, the DCLG index goes back to 1968, and though the average growth rates are lower in the longer period, the average difference across seasons is still very high at above 8 percent.[14])

[14] Results are available from the authors. We start in 1983 for comparability across regions.

Figure 1: Average annualized house price increases in summers and winters. DCLG 1983-2007.



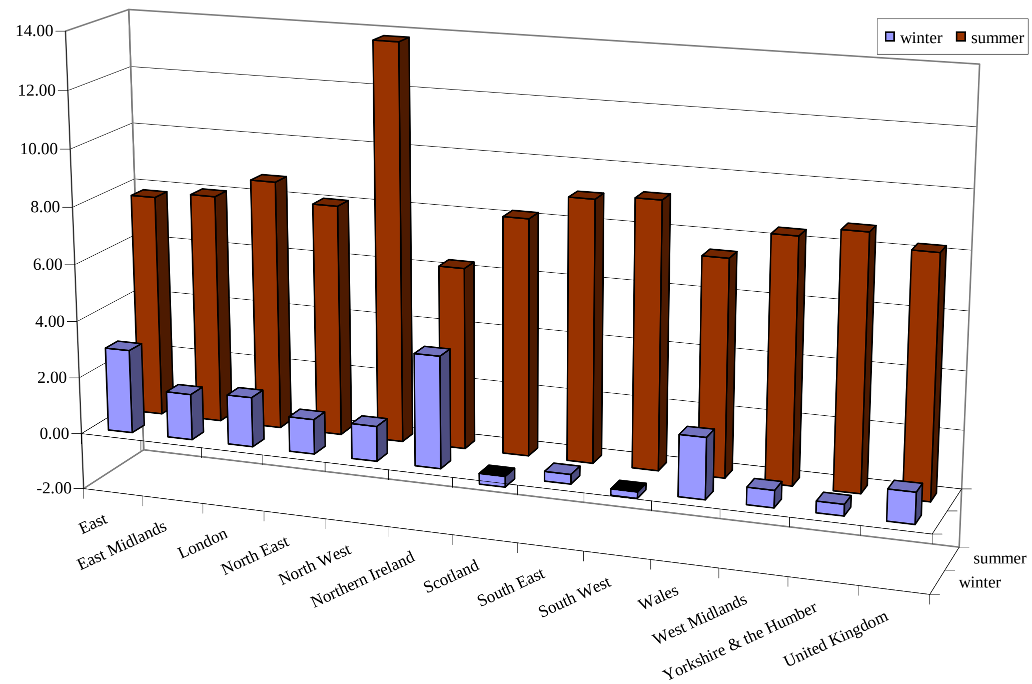
Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DCLG, 1983-2007.

The patterns are qualitatively similar when we use the Halifax index, not reported here in the interest of space (results are available from the authors). The annualized average price growth during the summer term is above 11 percent in all regions, with the exception of the North East and West Midlands, whereas the increase during the winter term is systematically below 5 percent, except for the North East region and London, where the increase is just above 5 percent. The average difference in growth rates across seasons is 7.4 percent. There are some non-negligible quantitative differences between the two sources, which might be partly explained by differences in coverage and by the lag between approval and completion, which, as mentioned, is one important difference between the two indexes. The two sources, however, point to a similar pattern of prices surging in the summer and stagnating in the winter.

The previous discussion was based on the seasonal pattern of nominal house prices. The seasonal pattern of real house prices (that is, house prices relative to the overall NSA price index) depends also on the seasonality of overall inflation. In the UK, overall price inflation displays some seasonality. The difference in overall inflation rates across the two seasons, however, can hardly "undo" the differences in nominal house price inflation, implying a significant seasonal also in real house prices. (See Figure 2.) Netting out the effect of overall inflation reduces the differences in growth rates between winters and summers to a country-wide average of 7.3 percent using the DCLG series and

5.6 using the Halifax series. We also looked at more disaggregated data, distinguishing between first-time buyers and former-owner occupiers, as well as purchases of new houses versus existing houses. Seasonal patterns are similar across the various groups; in the interest of space, we do not report the results here, but they are available upon request.

Figure 2: Average annualized real house price increases in summers and winters.
DCLG 1983-2007

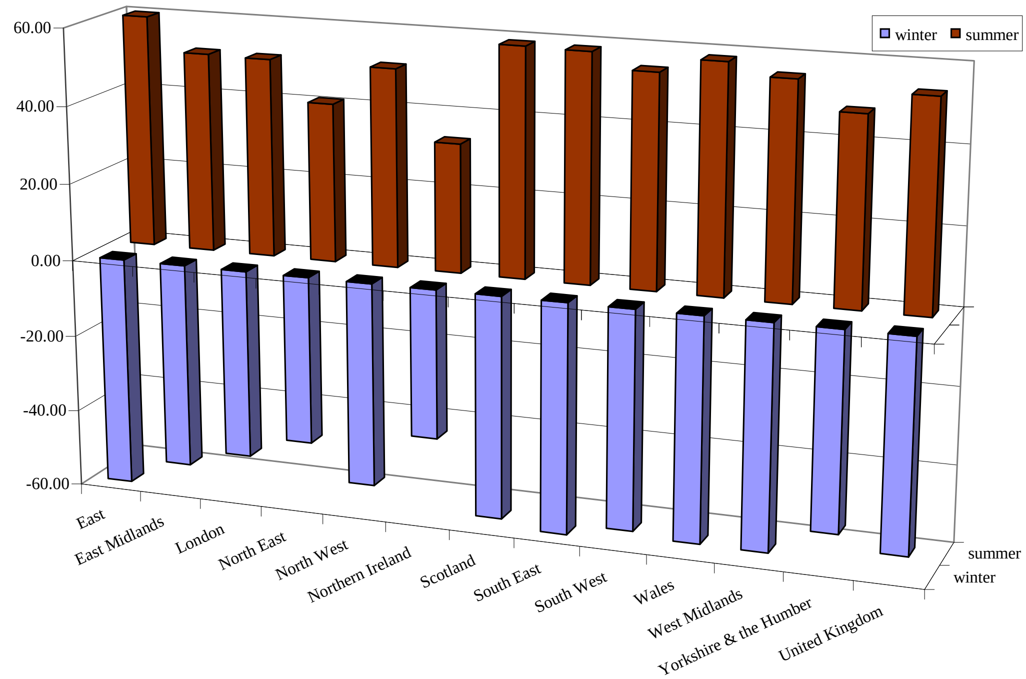


Note: Annualized real price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DCLG, 1983-2007.

Number of Transactions

Seasonal fluctuations in house prices are accompanied by qualitatively similar fluctuations in the number of transactions, proxied here by the number of mortgages. For comparability with the price sample, Figure 3 shows the growth rates in the number of mortgages in the two seasons from 1983 to 2007. (The data, which are compiled by CML, go back to 1974 for some regions; the patterns are qualitatively similar in the earlier period.) As the Figure shows, the number of transactions increases sharply in the summer term and accordingly declines in the winter term.

Figure 3: Average annualized increases in the number of transactions in summers and winters. CML 1983-2007



Note: Annualized growth rates in the number of transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. CML, 1983-2007.

Statistical Significance of the Differences between Summers and Winters

We test the statistical significance of the differences in growth rates across seasons, $\left[\ln\left(\frac{P_S}{P_W}\right)^2 - \ln\left(\frac{P_{W'}}{P_S}\right)^2\right]$, using a t-test on the equality of means.[15] Tables 1 through 3 report the average difference in growth rates across seasons and standard errors, together with the statistical significance. In particular, Table 1 reports the results for prices, both nominal and real, for all regions, using the data from DCLG and Table 2 shows the corresponding information using Halifax. Table 3 shows the differences in transactions' growth rates.

[15] The test on the equality of means is equivalent to the t-test on the slope coefficient from a regression of annualized growth rates on a dummy variable that takes value 1 if the observation falls on the second and third quarter and 0 otherwise. The dummy coefficient captures the annualized difference across the two seasons, regardless of the frequency of the data (provided growth rates are annualized). To see this note that the annualized growth rate in, say, the hot season, $\ln\left(\frac{P_S}{P_W}\right)^2$, is equal to the average of annualized quarterly growth rates in the summer term: $\ln\left(\frac{P_S}{P_W}\right)^2 = 2\ln\left(\frac{P_3}{P_1}\right) = \frac{1}{2}\left[4\ln\left(\frac{P_3}{P_2}\right) + 4\ln\left(\frac{P_3}{P_2}\right)\right]$, where the subindices indicate the quarter, and, correspondingly, $2\ln\left(\frac{P_{1'}}{P_3}\right) = \frac{1}{2}\left[4\ln\left(\frac{P_{1'}}{P_4}\right) + 4\ln\left(\frac{P_4}{P_3}\right)\right]$. Hence a regression with quarterly (or semester) data on a summer dummy will produce an unbiased estimate of the average difference in growth rates across seasons. We use quarterly data to exploit all the information and gain on degrees of freedom.

Table 1: Difference in annualized percentage changes in (nominal and real) house prices between summers and winters in the UK, by region. DCLG.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	6.536*	(3.577)	4.870	(3.461)
East Midlands	8.231**	(3.148)	6.408**	(3.131)
Gr. London	8.788***	(3.273)	6.966**	(3.372)
North East	8.511**	(3.955)	6.845*	(3.915)
North West	13.703***	(3.323)	12.583***	(3.245)
Northern Ireland	4.237	(3.431)	2.415	(3.467)
Scotland	10.393***	(2.793)	8.571***	(2.711)
South East	10.375***	(3.496)	8.709**	(3.301)
South West	11.244***	(3.419)	9.422***	(3.459)
Wales	7.180**	(3.504)	5.358	(3.442)
West Midlands	9.623***	(3.089)	7.801**	(3.070)
Yorkshire & the Humber	10.148***	(3.114)	8.325***	(3.056)
United Kingdom	9.008***	(2.304)	7.185***	(2.314)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Department of Communities and Local Government.

Table 2: Difference in annualized percentage changes in (nominal and real) house prices between summers and winters in the UK, by region. Halifax.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	9.885***	(3.604)	8.081**	(3.706)
East Midlands	10.247***	(3.393)	8.444**	(3.413)
Gr. London	5.696*	(3.048)	3.892	(3.221)
North East	2.197	(2.945)	0.394	(2.864)
North West	8.019***	(2.653)	6.216**	(2.548)
Northern Ireland	6.053*	(3.409)	4.25	(3.494)
Scotland	9.334***	(2.320)	7.530***	(2.272)
South East	7.104**	(3.019)	5.301*	(3.149)
South West	9.258**	(3.474)	7.454**	(3.549)
Wales	7.786**	(3.329)	5.983*	(3.288)
West Midlands	5.987*	(3.540)	4.183	(3.505)
Yorkshire & the Humber	7.253**	(2.892)	5.450*	(2.825)
United Kingdom	7.559***	(2.365)	5.756**	(2.400)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Halifax.

Table 3: Difference in annualized percentage changes in the volume of transactions between summers and winters in the UK, by region. CML.

Region	Difference	Std. Error
East Anglia	119.420***	(11.787)
East Midlands	104.306***	(11.151)
Gr. London	99.758***	(11.577)
North East	84.069***	(9.822)
North West	103.525***	(8.963)
Northern Ireland	71.466***	(12.228)
Scotland	116.168***	(9.843)
South East	117.929***	(9.710)
South West	110.996***	(8.764)
Wales	115.900***	(13.850)
West Midlands	112.945***	(9.496)
Yorkshire & the Humber	98.904***	(8.192)
United Kingdom	107.745***	(8.432)

Note: The Table shows the average differences (and standard errors) by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%.
Source: Council of Mortgage Lenders.

The differences in price increases across seasons are quite sizable for most regions, in the order of 7 to 9 percent on average in nominal terms (depending on whether DCLG or Halifax data are used) and 5.7 to 7 percent in real terms; the results from DCLG appear more significant than those from Halifax from a statistical point of view. For transactions, the differences reach 108 percent for the country as a whole. Put together, the data point to a strong seasonal cycle in virtually all regions, with a large increase in transactions and prices during the summer relative to the winter term.

Rents and Mortgage Rates Data on rents are not well documented. Only in recent years have data collection efforts started, but there is no long enough time-series to detect seasonality.[16] One source that can serve at least as indicative, is the average registered private rents collected by the UK Housing and Construction Statistics; the data run on a quarterly basis from 1979:01 to 2001:04. We run regressions using as dependent variables both the rent levels and the log of rents on a dummy variable taking value 1 in the second and third quarters and 0 otherwise, detrending the data in different ways. The data showed no deterministic seasonality (regression outcomes available from the authors). This is in line with anecdotal evidence suggesting that rents are fairly sticky. Given the paucity of data on rents, there is little we can say with high confidence. Still, note that for rents to be the driver of price seasonality, one would need an enormous degree of seasonality in rents (as well as a high discount rate), since prices should in principle, according to the standard asset-pricing approach, reflect the present values of all future rents (in other words, prices should be

[16] See new data produced by the Chartered Institute of Housing since 1999 and ONS since 1996.

less seasonal than rents). The lack of even small discernible levels of seasonality in the data suggests that we need alternative explanations for the observed seasonality in prices.

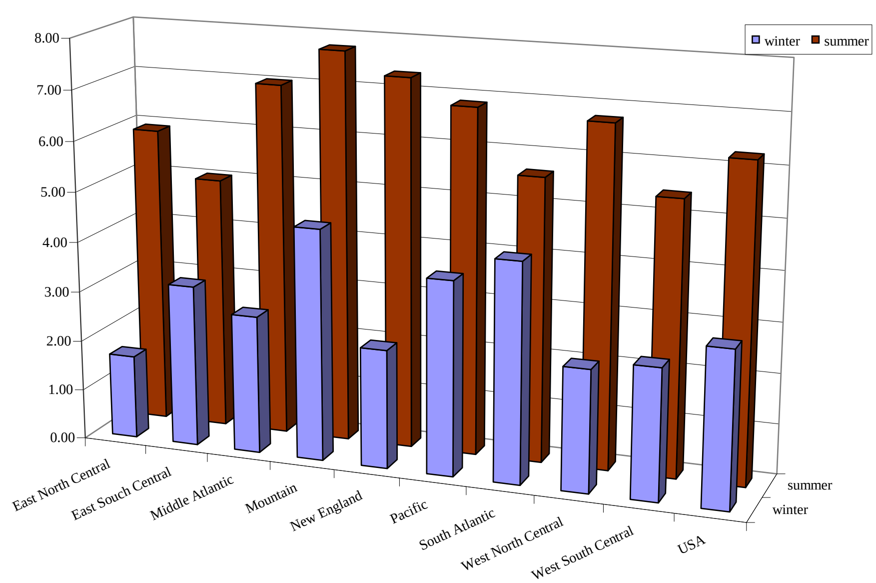
Interest rates in the UK do not exhibit a seasonal pattern, at least in the last four decades of data. We investigated seasonality in different interest rate series provided by the Bank of England: The repo (base) rate; an average interest rate charged by the four UK major banks—before the crisis (Barclays Bank, Lloyds Bank, HSBC, and National Westminster Bank); and a weighted average standard variable mortgage rate from banks and Building Societies. None of the interest rate series displays seasonality (results available from the authors).

Housing Market Seasonality in the US

Nominal and Real House Prices

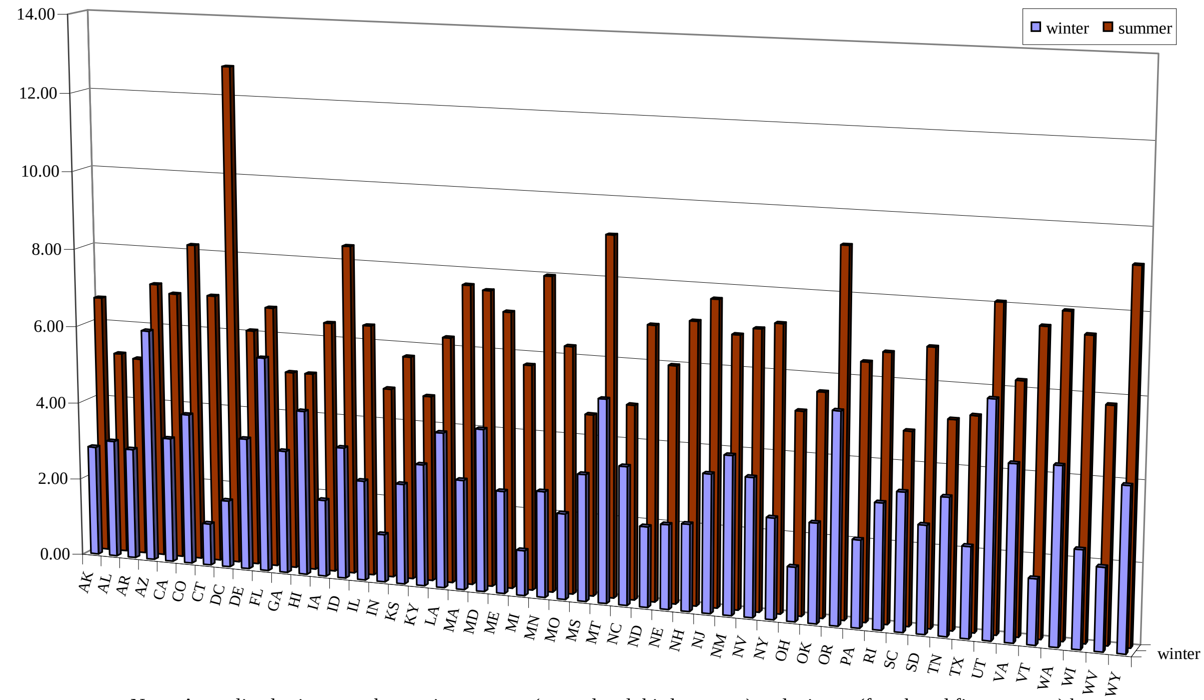
Figure 4 illustrates the annualized nominal house price increases for different regions from OFHEO. Figure 5 shows the corresponding plot for different states, also from OFHEO, and Figure 6 shows the plot using the S&P's Case-Shiller indexes for major cities. One first observation is that for most US regions, the seasonal pattern is qualitatively similar to that in the UK, albeit the extent of seasonality is generally smaller. For some of the US major cities, however, the degree of seasonality is comparable to that in the UK.

Figure 4: Average annualized house price increases in summers and winters, by region. OFHEO 1991-2007



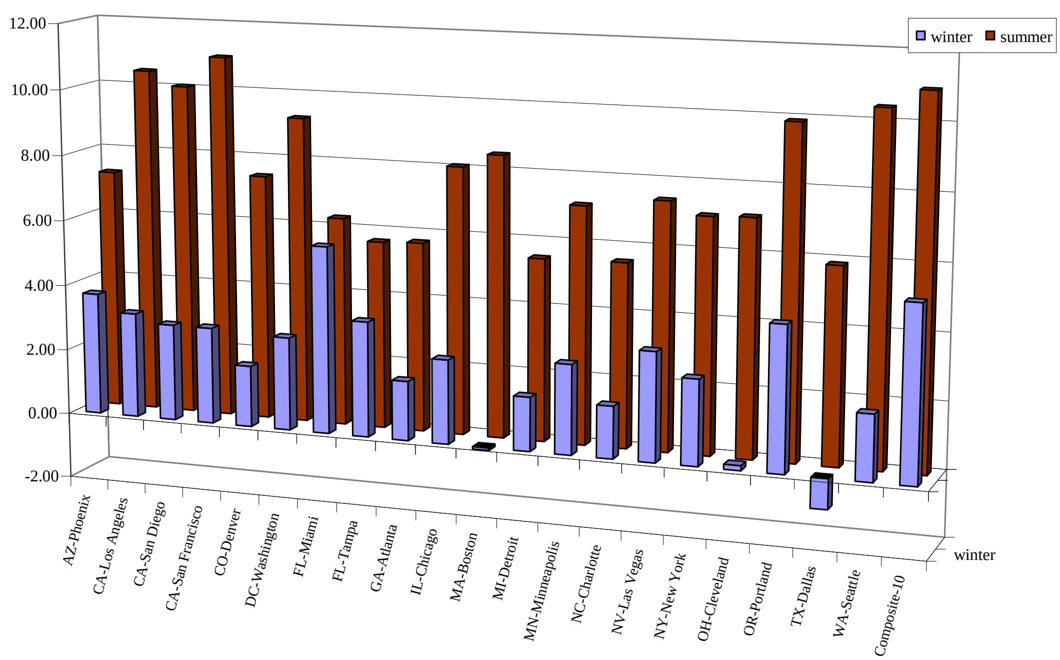
Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. OFHEO, 1991-2007.

Figure 5: Average annualized house price increases in summers and winters by state. OFHEO 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. state. OFHEO, 1991-2007.

Figure 6: Average annualized house price increases in summers and winters by city. S&P's Case-Shiller 1987-2007

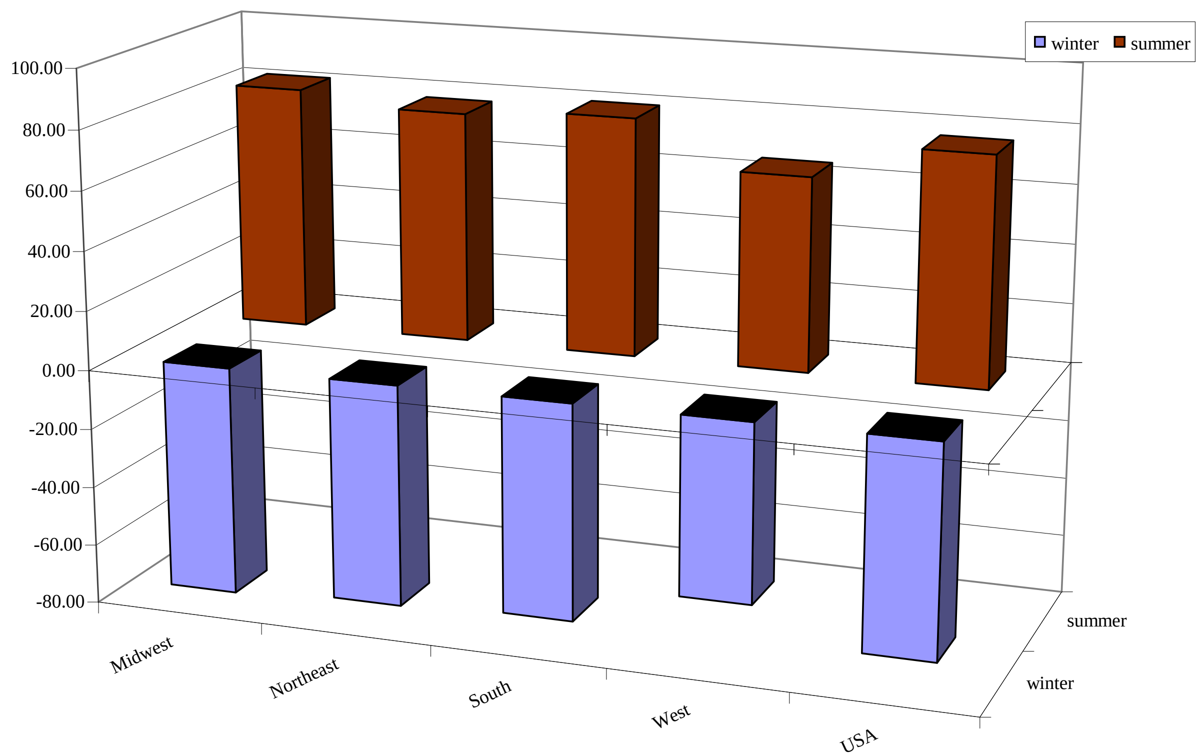


Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. city. S&P Case and Shiller, 1987-2007.

The results using real prices (in terms of differences between seasons) are virtually identical to the ones for nominal prices, as CPI inflation rates hardly differ across seasons over the period analyzed and hence the differences in real growth rates across seasons are almost identical to the differences in nominal growth rates. These differences are later summarized in Table 4. (Figures are omitted in the interest of space, but are available from the authors).

Transactions Figure 7 shows the annualized growth rates in the number of transactions from 1991 through to 2007 for main Census regions; the data come from NAR.[17] Seasonality in transactions is overwhelming: The volume of transactions rises sharply in the summer and falls in the winter, by even larger magnitudes than in the UK.

Figure 7: Average annualized increases in the number of transactions in summers and winters. NAR 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. NAR 1991-2007.

Statistical Significance of the Differences between Summers and Winters We summarize the differences in growth rates across seasons and report the results from a test on mean

[17] The series actually starts in 1989, but we use 1991 for comparability with the OFHEO-census-level division price series; adding these two years does not change the results.

differences in Tables 4 through 7. Table 4 shows the results for prices using OFHEO's Census-division level; Table 5 shows the results using OFHEO's state-level data; Table 6 shows the results using S&P's Case-Shiller city-level data; and Table 7 shows the results for transactions from NAR.

Regarding house prices, for the US as a whole, the differences in annualized growth rates (nominal and real) are in the order of 3 percent. There is considerable variation across regions, with some displaying virtually no seasonality (South Atlantic) and others (East and West North Central, New England and Middle Atlantic) displaying significant levels of seasonality. This variability becomes more evident at the state level. Interestingly, the Case-Shiller index for cities displays even higher levels of seasonality, comparable to the levels observed in UK regions. (This will be consistent with our model, which, ceteris paribus, generates more seasonality when the bargaining power of sellers is higher, as it is likely to be the case in cities, where land is relatively scarce.)

The volume of transactions is extremely seasonal in the US, even more than in the UK, with an average difference in growth rates across seasons of 148 percent and the pattern is common to all regions.

Table 4: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) in the US, by region

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East North Central	4.262***	(0.772)	4.106***	(0.924)
East South Central	1.811***	(0.535)	1.654**	(0.701)
Middle Atlantic	4.273**	(1.619)	4.116**	(1.660)
Mountain	3.166**	(1.205)	3.009**	(1.281)
New England	4.980**	(2.081)	4.823**	(2.181)
Pacific	3.010	(2.117)	2.853	(2.195)
South Atlantic	1.281	(1.277)	1.125	(1.370)
West North Central	4.333***	(0.743)	4.176***	(0.872)
West South Central	2.836***	(0.537)	2.679***	(0.650)
USA	3.169***	(0.967)	3.012***	(1.081)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: OFHEO Purchase-only Index.

Table 5: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) by US state.

	Nominal house price		Real house price	
State	Difference	Std. Error	Difference	Std. Error
Alabama	3.812**	(1.400)	3.655**	(1.378)
Alaska	2.189***	(0.692)	2.032**	(0.848)
Arizona	2.263**	(0.848)	2.106**	(0.950)
Arkansas	1.109	(2.586)	0.953	(2.583)
California	3.656	(3.398)	3.499	(3.479)
Colorado	4.285***	(1.323)	4.129***	(1.447)
Connecticut	5.819***	(2.055)	5.662**	(2.133)
District of Columbia	11.040**	(4.229)	10.883**	(4.150)
Delaware	2.687	(1.862)	2.530	(1.925)
Florida	1.185	(2.525)	1.028	(2.571)
Georgia	1.921**	(0.743)	1.764*	(0.887)
Hawaii	0.850	(3.668)	0.693	(3.677)
Idaho	4.440***	(0.615)	4.283***	(0.711)
Illinois	5.035***	(1.659)	4.878***	(1.688)
Indiana	3.864***	(0.755)	3.707***	(0.859)
Iowa	3.621***	(0.768)	3.464***	(0.884)
Kansas	3.134***	(0.709)	2.977***	(0.925)
Kentucky	1.623***	(0.570)	1.466**	(0.707)
Louisiana	2.300***	(0.827)	2.143**	(0.921)
Maine	4.823**	(2.219)	4.666*	(2.339)
Maryland	3.384	(2.341)	3.227	(2.396)
Massachusetts	4.407**	(2.146)	4.250*	(2.231)
Michigan	4.573***	(1.568)	4.416**	(1.698)
Minnesota	5.290***	(1.376)	5.133***	(1.484)
Missouri	4.085***	(0.646)	3.929***	(0.758)
Mississippi	1.379	(1.028)	1.222	(1.108)
Montana	3.957**	(1.469)	3.800**	(1.510)
North Carolina	1.417**	(0.641)	1.260	(0.764)
North Dakota	4.908***	(1.353)	4.751***	(1.423)
Nebraska	3.842***	(1.082)	3.685***	(1.162)
New Hampshire	4.918**	(2.391)	4.761*	(2.463)
New Jersey	4.197*	(2.076)	4.041*	(2.126)
New Mexico	2.857*	(1.560)	2.700	(1.623)
Nevada	3.540	(2.946)	3.383	(3.026)
New York	4.662**	(1.815)	4.505**	(1.872)
Ohio	3.729***	(0.731)	3.572***	(0.911)
Oklahoma	3.095***	(0.477)	2.938***	(0.511)
Oregon	3.903***	(1.380)	3.746***	(1.310)
Pennsylvania	4.226***	(1.317)	4.069***	(1.329)
Rhode Island	3.544	(2.842)	3.388	(2.969)
South Carolina	1.360*	(0.698)	1.203	(0.771)
South Dakota	4.201***	(1.171)	4.044***	(1.248)
Tennessee	1.759**	(0.685)	1.602*	(0.834)
Texas	3.045***	(0.674)	2.888***	(0.763)
Utah	2.204	(1.820)	2.047	(1.803)
Virginia	1.873	(1.758)	1.716	(1.835)
Vermont	5.945**	(2.430)	5.788**	(2.373)
Washington	3.563**	(1.377)	3.406**	(1.377)
Wisconsin	5.007***	(0.738)	4.850***	(0.848)
West Virginia	3.753**	(1.702)	3.596**	(1.765)
Wyoming	5.091***	(1.365)	4.935***	(1.391)

Note: The Table shows the average differences (and standard errors), by state for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: OFHEO Purchase-only Index.

Table 6: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) by US city.

	Nominal house price		Real house price	
City	Difference	Std. Error	Difference	Std. Error
AZ-Phoenix	3.571	(3.307)	3.405	(3.357)
CA-Los Angeles	7.273**	(3.478)	6.884*	(3.535)
CA-San Diego	7.107**	(3.204)	6.717**	(3.275)
CA-San Francisco	8.051**	(3.009)	7.662**	(3.045)
CO-Denver	5.576***	(1.599)	5.186***	(1.805)
DC-Washington	6.439**	(2.604)	6.050**	(2.645)
FL-Miami	0.636	(2.744)	0.246	(2.838)
FL-Tampa	2.171	(2.384)	1.781	(2.484)
GA-Atlanta	3.920***	(0.903)	3.763***	(1.042)
IL-Chicago	5.530***	(1.342)	5.141***	(1.459)
MA-Boston	8.560***	(2.091)	8.170***	(2.325)
MI-Detroit	3.864*	(1.909)	3.707*	(2.060)
MN-Minneapolis	4.431***	(1.528)	4.265**	(1.741)
NC-Charlotte	3.968***	(0.721)	3.578***	(0.836)
NV-Las Vegas	4.149	(3.216)	3.76	(3.262)
NY-New York	4.477**	(2.161)	4.087*	(2.342)
OH-Cleveland	6.942***	(0.973)	6.553***	(1.041)
OR-Portland	5.551***	(1.485)	5.161***	(1.388)
TX-Dallas	6.776***	(1.380)	6.138***	(1.823)
WA-Seattle	8.437***	(1.953)	8.175***	(1.942)
Composite-20 cities	6.051***	(2.227)	5.662**	(2.344)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: SP's Case-Shiller index.

Table 7: Difference in annualized percentage changes in house transactions between semesters (second-third quarters vis-à-vis fourth-first quarters) by US region.

Region	Coef.	Std. Error
Midwest	159.473***	(6.488)
Northeast	152.551***	(4.918)
South	153.009***	(4.702)
West	124.982***	(6.312)
United States	148.086***	(5.082)

Note: The Table shows the average differences (and standard errors) by region for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: National Association of Realtors.

Rents and Mortgage Rates As was the case for the UK, the paucity of rent data for the US is regrettable. The Bureau of Labor Statistics (BLS) provides two series that can serve as proxies: One is the NSA series of owner's equivalent rent and the second is the NSA rent of primary residence; both series are produced for the construction of the CPI and correspond to averages over all cities.

For each series, we run regressions using as dependent variables both the rent levels and the log of rents, de-trended in various ways, on a summer-term dummy. The results (available from the authors) as for the US, yielded no discernible pattern of seasonality. We take this as only suggestive as, of course, the data are not as clean and detailed as we would wish. To reiterate, however, if seasonality in rents were the driver of seasonality in prices, we should observe enormous seasonality in rental flows in order to justify the observed seasonality in house prices. In the model we present later, we will work under the constraint that rents are aseasonal.

As first documented by Barsky and Miron (1989), interest rates in recent decades do not exhibit seasonality. We investigated in particular data on mortgage rates produced by the Board of Governors of the Federal Reserve, corresponding to contract interest rates on commitments for fixed-rate first mortgages; the data are quarterly averages beginning in 1972; the original data are collected by Freddie Mac. Consistent with the findings of Barsky and Miron (1989) and the evidence from the UK, we did not find any significant deterministic seasonality. (Results available from the authors.)

2.3 Cross-market comparisons and market thickness

The data description makes it evident that seasonal cycles are present across most of the US and the UK, although with some heterogeneity with regards to intensity. In particular, though most cities in the US display strong seasonality, Miami and Tampa show little (and statistically insignificant) variation over the season. Given the data limitations (20 observations on price seasonality corresponding to the cities in Case-Shiller data or 50 observations, when using OFHEO state-level data), it would be virtually impossible to draw causal links from the potential triggers of seasonality, e.g., i) winters are mild in these cities and ii) there is a larger population of elder people, both of which are in turn intimately related. We note, though, that the mildness of a winter per se does not straightforwardly predict a-seasonality, since cities such as Los Angeles, San Diego, or San Francisco display strong seasonality in prices, despite their benign weather. More generally, it is unlikely that most people would be willing to pay a significant part of their wealth for the convenience of searching under good weather. Similarly, and as noted earlier, only a small portion of the population of movers has school-age children. The key conceptual point of the model is that even slight differences in the "fundamentals" of the seasons have the potential to trigger thick-market effects with large swings in the volume of transactions and prices.

We also note that US cities tend to display more seasonality than the US as a whole, a pattern that, as we shall explain, can be rationalized by our model. (We further discuss this issue in Section

4.3.2.) Some may argue that cities by their sheer size, are likely to be "thicker" throughout the year and hence seasonal differences in thickness are relatively unimportant. Anecdotal evidence, however, suggests that even within cities, housing markets are highly segmented, as people tend to search in relatively narrow neighborhoods and geographical areas (e.g., to be close to school, jobs, families). Thus, for example, London or DC as a whole are not the relevant sizes of the market, and it would be improper to use them as boundaries to define market thickness (e.g., for those familiar with London's geography and social structure, people searching in South Kensington will never search in the East End). In other words, seemingly large cities may mask a collection of relatively smaller and segmented housing markets that can see significant changes in thickness throughout the year. A limitation of the data is hence that we cannot meaningfully compare thickness across cities or states.

3 A Search-and-Matching Model for the Housing Market

We have argued before that the predictability and size of the seasonal variation in house prices pose a challenge to models of the housing market relying on a standard asset-market approach. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal service cost. In Appendix 7.2 we assess the extent to which seasonality in service flows might be driving seasonality in prices. The exercise makes clear that a standard asset-pricing approach that relies on standard (perfect) arbitrage leads to implausibly large levels of required seasonality in service flows.

Our findings suggest that there are important frictions in the market that impair the ability of investors to gain from seasonal arbitrage and therefore call for a deviation from the standard asset-pricing approach.[18] But perhaps a more fundamental reason to deviate is the overwhelming evidence that buying and selling houses involve a non-trivial search process that is not well captured in the standard asset-pricing approach. Furthermore, as it is also the case in labor markets (and largely the motivation for the labor-search literature) the coexistence at any point in time of a stock of vacant houses and a pool of buyers searching for houses, suggests a lack of immediate market clearing; explicitly modelling the frictions that impair clearing can help in the understanding of housing market fluctuations. In this Section we develop a search-and-matching model for the housing market that

[18] The need to deviate from asset-market approach has been acknowledge in previously, see e.g. Stein (1995) and Ortalo-Magne and Rady (2005).

contains two elements that can account for seasonality (and, likely, lower frequency fluctuations): "match-specific quality" and "thick-market effects".

3.1 The Economy

The economy is populated by a unit measure of infinitely lived agents, who have linear preferences over housing services and a non-durable consumption good. Each period agents receive a fixed endowment of the consumption good which they can either consume or use to buy housing services. An agent can only enjoy housing services from living in one house at a time, that is, he can only be "matched" to one house at a time. Agents who are not matched to a house seek to buy one ("buyers").

There is a unit measure of housing stock. Correspondingly, each period a house can be either matched or unmatched. A matched house delivers a flow of housing services of quality ε to its owner. The quality of housing services ε is match-specific, and it reflects the suitability of a match between a house and its homeowner. In other words, for any house, the quality of housing services is idiosyncratic to the match between the house and the potential owner. For example, a particular house may match a buyer's taste perfectly well, while at the same time being an unsatisfactory match to another buyer. Hence, ε is not the type of house (or of the seller who owns a particular house). This is consistent with our data, which are adjusted for houses' characteristics, such as size and location, but not for the quality of a match.[19]

We assume that in a market with many houses for sale, a buyer is more likely to find a better match, what we refer to as "thick-market effect.". As in Diamond (1981), we model this idea by assuming that the match-specific quality ε follows a distribution $F(\varepsilon, v)$, with positive support and finite mean, and

$$F(., v') \leq F(., v) \Leftrightarrow v' > v, \tag{1}$$

where v denotes the stock of vacant houses. In words, when the stock of houses v is larger, a random match-quality draw from $F(\varepsilon, v)$ is likely to be higher.[20]

[19] Neither repeat-sale indices nor hedonic price indexes can control for the quality of a match, which is not observed by data collectors.

[20] One way to interpret our assumption is to think of order statistics. Suppose the buyer samples n units of vacant houses when the stock of vacancies is v. As long as the number of units sampled n increases in v, the maximum match quality ε in the sample will be "stochastically larger." In other words, for any underlying distribution of match quality, the distribution of the maximum in a sample of size n will first-order stochastically dominate the distribution of the maximum in a smaller sample $n' < n$. As such, F can be interpreted as the distribution of the sample maximum.

Unmatched houses are "for sale" and are owned by "sellers;" sellers receive a flow u from any unmatched house they own, where the flow u is common to all sellers.

3.2 Seasons and Timing

There are two seasons, $j = s, w$ (for summer and winter); each model period is a season, and seasons alternate. At the beginning of a period j, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, and the house is put up for sale, adding to the stock of vacant houses, denoted by v^j. The homeowner whose match has broken becomes simultaneously a seller and a buyer, adding to the pool of buyers, denoted by b^j. In our baseline model, the parameter ϕ^j is the only (ex ante) difference between the seasons.[21] We focus on periodic steady states with constant v^s and v^w. Since a match is between one house and one agent, and there is a unit measure of agents and a unit measure of houses, it is always the case that the mass of vacant houses equals the mass of buyers: $v^j = b^j$.

Our objective is to investigate how such deterministic driver of seasonality can be amplified and revealed as seasonality in transactions and prices in the housing market due to the thick-market-effects on the match-specific quality. By focusing on the periodic steady state, we are studying a deterministic cycle and agents are aware that they are in such a cycle with ϕ^j, transactions and prices fluctuating between high and low across the two seasons.

Each period, every buyer meets with a seller and every seller meets with a buyer. Upon meeting, the match-specific quality between the buyer and the house is drawn from a distribution $F(\varepsilon, v)$. If they agree on a transaction, the buyer pays a price (discussed later) to the seller, and starts enjoying the housing services ε. If not, the buyer looks for a house again next period, the seller receives the flow u, and puts the house up for sale again next period.[22] An agent can hence be either a matched homeowner or a buyer, and, at the same time, he could also be a seller. Also, sellers may have multiple houses to sell.

[21] This difference could be determined, for example, by the school calendar or summer marriages, among other factors, exogenous to our model. In Section 5.2, we discuss seasonal transaction costs as an alternative driver of seasonality.

[22] In Section 5.2 we relax the assumption that if the transaction does not go through, buyer and seller need to wait for next period to transact with other agents.

3.3 The Homeowner

To study pricing and transaction decisions, we first derive the value of living in a house with match quality ε starting in season s is given by:

$$H^s(\varepsilon) = \varepsilon + \beta\phi^w H^w(\varepsilon) + \beta(1-\phi^w)[V^w + B^w],$$

where $\beta \in (0,1)$ is the discount factor. With probability $(1-\phi^w)$ he receives a moving shock and becomes both a buyer and a seller (putting his house up for sale), with continuation value (V^w+B^w), where V^j is the value of a vacant house to its seller and B^j is the value of being a buyer in season $j = s, w$, defined later. With probability ϕ^w he keeps receiving housing services of quality ε and stays in the house. The formula for $H^w(\varepsilon)$ is perfectly isomorphic to $H^s(\varepsilon)$; in the interest of space we omit here and throughout the paper the corresponding expressions for season w. The value of being a matched homeowner can be therefore re-written as:

$$H^s(\varepsilon) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon + \frac{\beta(1-\phi^w)(V^w+B^w) + \beta^2\phi^w(1-\phi^s)(V^s+B^s)}{1-\beta^2\phi^w\phi^s}, \tag{2}$$

which is strictly increasing in ε. The first term that enters the housing value $H^s(\varepsilon)$ is the effective (adjusted for moving probabilities) present discounted value of staying in a house with match quality ε and the second term contains the values in the event that the match may dissolve in any future summers or winters.

3.4 Market Equilibrium

We focus on the case in which both seller and buyer observe the quality of the match, ε, which is drawn from $F^j(\varepsilon) \equiv F(\varepsilon, v^j)$; we derive the results for the case in which the seller cannot observe ε in Appendix 7.5. If the transaction goes through, the buyer pays a mutually agreed price to the seller, and starts enjoying the housing services flow in the same season j. If the transaction does not go through, the buyer receives zero housing services and looks for a house again next season. This will be the case, for example, if buyers searching for a house pay a rent equal to the utility they derive from the rented property; what is key is that the rental property is not owned by the same potential seller with whom the buyer meets. On the seller's side, when the transaction does not go through, he receives the flow u in season j and puts the house up for sale again next season. The flow u can be interpreted as a net rental income received by the seller. Again, what is key is that the tenant is not the same potential buyer who visits the house.

3.4.1 Reservation Quality

The total surplus of a transaction is:

$$S^s(\varepsilon) = H^s(\varepsilon) - [\beta(B^w + V^w) + u]. \tag{3}$$

Intuitively, a new transaction generates a new match of value $H^s(\varepsilon)$; if the transaction does not goes through, the buyer and the seller obtain βB^w and $(\beta V^w + u)$, respectively. Since ε is observable and the surplus is transferrable, a transaction goes through as long as the total surplus $S^s(\varepsilon)$ is positive. Given $H^s(\varepsilon)$ is increasing in ε, a transaction goes through if $\varepsilon \geqslant \varepsilon^s$, where the reservation ε^s is defined by:

$$\varepsilon^s =: H^s(\varepsilon^s) = \beta(B^w + V^w) + u, \tag{4}$$

and $1 - F^s(\varepsilon^s)$ is thus the probability that a transaction is carried out. Since the reservation quality ε^s is related to the total surplus independently of how the surplus is divided between the buyer and the seller, we postpone the discussion of equilibrium prices to Section 4.2. Using the expression of housing value $H^s(\varepsilon)$ in (2), equation (4) becomes:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s = u - \frac{\beta^2\phi^w(1-\phi^s)}{1-\beta^2\phi^w\phi^s}(B^s + V^s) + \frac{1-\beta^2\phi^s}{1-\beta^2\phi^w\phi^s}\beta\phi^w(B^w + V^w). \tag{5}$$

The Bellman equation for the sum of values is:

$$B^s + V^s = \beta(B^w + V^w) + u + [1 - F^s(\varepsilon^s)]\, E^s[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], \tag{6}$$

where $E^s[.]$ indicates the expectation is taken with respect to distribution $F^s(.)$. The sum of values in season s covers the outside option, $\beta(B^w + V^w) + u$ (the flow u plus the option value of buying and selling next season) and, with probability $[1 - F^s(\varepsilon^s)]$, on the expected surplus from a transaction for sellers and buyers. Solving this explicitly and using the expression for $S^j(\varepsilon)$, $j = s, w$ in (20):

$$B^s + V^s = \frac{u}{1-\beta} + \frac{(1+\beta\phi^w)\, h^s(\varepsilon^s) + \beta(1+\beta\phi^s)\, h^w(\varepsilon^w)}{(1-\beta^2)(1-\beta^2\phi^w\phi^s)}, \tag{7}$$

where $h^s(\varepsilon^s) \equiv [1 - F^s(\varepsilon^s)]\, E[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s]$ is the expected surplus of quality above threshold ε^s.

The equilibrium values $\varepsilon^s, \varepsilon^w, (B^s + V^s)$, and $(B^w + V^w)$ in (5) and (7) depend on equilibrium vacancies v^s and v^w, which we now derive.

3.4.2 Stock of vacant houses

In any season s, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$v^s = (1-\phi^s)[v^w(1 - F^w(\varepsilon^w)) + 1 - v^w] + v^w F^w(\varepsilon^w)$$

where the first term corresponds to houses that received a moving shock and hence were put for sale this season and the second term corresponds to vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = 1 - \phi^s + v^w F^w (\varepsilon^w) \phi^s. \tag{8}$$

The equilibrium quantities $(B^s + V^s, B^w + V^w, \varepsilon^s, \varepsilon^w, v^s, v^w)$ jointly satisfy equations (5), (7), and (8) together with the isomorphic equations for the other season. They are independent of how the total surplus is shared across buyers and sellers, that is independent of the exact price-setting mechanism. We hence discuss seasonality in vacancies and transactions first, before we specify the particular price-setting mechanism.

4 Model-generated Seasonality

The driver for seasonality in the baseline model is the higher moving probability in the summer: $1-\phi^s > 1-\phi^w$. As shown earlier the equilibrium quantities $(B^s + V^s, B^w + V^w, \varepsilon^s, \varepsilon^w, v^s, v^w)$ jointly satisfy six equations. Before jumping directly to the quantitative results we discuss the underlying mechanisms through which a higher moving probability in the summer leads to a larger stock of vacancies and a higher expected return for buyers and sellers, i.e. $v^s > v^w$ and $B^s + V^s > B^w + V^w$. This Section is hence aimed at providing intuition for the mechanics of the model.

It is important to re-iterate that our notion of seasonality is not a cross-steady states comparison, that is, we are not comparing an steady state with high moving probability to another steady state with low moving probability. Instead, the seasonal values we derive are equilibrium values along a periodic steady state where agents take into account that the economy is fluctuating deterministically between the hot and the cold seasons.

Using (8), the stock of vacant houses in season s is given by:

$$v^s = \frac{1 - \phi^s + \phi^s F^w (\varepsilon^w) (1 - \phi^w)}{1 - F^s (\varepsilon^s) F^w (\varepsilon^w) \phi^s \phi^w}. \tag{9}$$

(The expression for v^w is correspondingly isomorphic). The ex ante higher probability of moving in the summer $(1-\phi^s > 1-\phi^w)$ clearly has a direct positive effect on v^s, and, as it turns out, this effect also dominates quantitatively when we calibrate the model to match the average duration of stay in a house.[23] Thus, we have $v^s > v^w$. The model predicts an almost one-to-one relationship between

[23] More specifically, the numerator is a weighted average of 1 and $F^w (\varepsilon^w) (1 - \phi^w)$, with $1 - \phi^s$ being the weight

the seasonality in the probability of moving $(1-\phi^s)/(1-\phi^w)$ and the seasonality in the stock of vacancy v^s/v^w. The probability of moving is exogenous in our model and we calibrate it so as to match the seasonality in vacancies. Our main interest is to predict the seasonality in transactions and prices.

To that aim, we first take a somewhat tedious but useful detour to comment on the seasonality of the sum of values $(B^j + V^j)$ and the reservation quality ε^j, $j = s, w$. Intuitively, a higher stock of vacancies in the summer implies higher expected returns to a buyer and a seller in the summer because of better matches through the thick-market effect. These higher expected returns in the summer, however, also raise the outside options of a buyer and a seller in the winter. Higher outside options make both the buyer and the seller more demanding and tend to increase the reservation quality in the winter. In equilibrium, however, the overall effect on reservation quality is ambiguous as we show more explicitly below.

The higher stock of vacancies in the summer, $v^s > v^w$, implies a higher expected surplus quality for any given cutoff through the thick-market effects as in (1). To see this rewrite $h^s(x) = \int_x [1 - F^s(\varepsilon)]\, d\varepsilon$ using integration by parts. Given $\phi^w > \phi^s$, it follows from equation (7) that $B^s + V^s > B^w + V^w$ if the two equilibrium cutoffs ε^s and ε^w are close. In other words, the expected return $(B^j + V^j)$ is higher in the summer as long as the thick-market-effect dominates a potentially offsetting equilibrium effect from the reservation quality. Using the definition of reservation quality in (4), lower outside options $(B^w + V^w)$ in the summer imply a lower housing value for the marginal transaction in the summer,[24]

$$H^s(\varepsilon^s) < H^w(\varepsilon^w). \tag{10}$$

However, this does not necessarily imply a lower reservation quality in the summer, $\varepsilon^s < \varepsilon^w$. This is because the ranking of the housing values in the two seasons, $H^s(\varepsilon)$ and $H^w(\varepsilon)$, depends on the level of ε. To see this explicitly, from (2), note that $H^j(\varepsilon)$ is linear in ε for $j = s, w$. Given $\phi^w > \phi^s$, $H^s(.)$ is steeper than $H^w(.)$. The difference in the intercepts between $H^s(.)$ and $H^w(.)$ is proportional to:

$$\beta\left[(1-\phi^w)(1-\beta\phi^s)(B^w+V^w) - (1-\phi^s)(1-\beta\phi^w)(B^s+V^s)\ \right],$$

assigned to 1 in the equation for v^s. Since $1-\phi^s > 1-\phi^w$, the equation for v^s assigns a higher weight on 1. Since $F^w(\varepsilon^w)(1-\phi^w) < 1$, higher weight on 1 leads to $v^s > v^w$; this is because $F^w(\varepsilon^w)(1-\phi^w)$ is virtually aseasonal as there are two opposite effects: $F^w(\varepsilon^w) > F^s(\varepsilon^s)$ and $(1-\phi^w) < (1-\phi^s)$ that tend to largely cancel each other.

[24] Note, though, that because of the thick-market effect, the average housing value will still be higher in the summer (even if the marginal value is lower).

which is negative when $B^s + V^s > B^w + V^w$.[25] Therefore, $H^s(.)$ and $H^w(.)$ must cross once at $\hat{\varepsilon}$. Thus if the equilibrium reservation quality in the summer is sufficiently high, $\varepsilon^s > \hat{\varepsilon}$, then $H^s(\varepsilon^s) > H^w(\varepsilon^s)$. Therefore, in order for inequality (10) to hold, we must have $\varepsilon^w > \varepsilon^s$. In this case, the intuition that a lower outside option in the summer leads to a lower cutoffs prevails. On the other hand, if the equilibrium reservation quality in the summer is sufficiently low, $\varepsilon^s < \hat{\varepsilon}$, then $H^s(\varepsilon^s) < H^w(\varepsilon^s)$; in this case, the inequality $\varepsilon^w > \varepsilon^s$ is no longer required for inequality (10) to hold. In sum, the two equilibrium cutoffs cannot be ranked. Quantitatively, the two cutoffs turn out to be close for reasonable parametrizations of the model.

4.1 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[1 - F^s(\varepsilon^s)\right]. \tag{11}$$

(An isomorphic expression holds for Q^w). From (11), it is evident that a bigger stock of vacancies in the summer, $v^s > v^w$, has a direct positive effect on the number of transactions in the summer relative to winter. Furthermore, if the probability of a transaction is also higher in the summer, then transactions will be more seasonal than vacancies. This amplification effect, which follows from the first-order stochastic dominance of $F^s(.)$ over $F^w(.)$, is indeed present in our quantitative exercise.[26] Intuitively, a higher stock of vacancies leads to better matches through the thick-market-effect, resulting in a higher transaction probability.

4.2 Seasonality in Prices

As discussed earlier, results on seasonality in vacancies and transactions are independent of the exact price-setting mechanism, i.e. how the surplus is shared across buyers and sellers.

Let $S_v^s(\varepsilon)$ and $S_b^s(\varepsilon)$ be the surpluses of a transaction to the seller and to the buyer, respectively,

[25] This is because $(1-\phi^w)(1-\beta\phi^s) - (1-\phi^s)(1-\beta\phi^w) = (1-\beta)(\phi^s - \phi^w) < 0$

[26] As said, there could be an additional effect if the cutoffs are highly seasonal. For example, if $\varepsilon^w > \varepsilon^s$, there will be even lower volume of transactions in the winter. This is because the outside option for both buyers and sellers is to wait and transact in the next season. Therefore, a higher outside option in the winter makes both buyers and sellers more demanding in the winter and hence less likely to transact, yielding an even smaller number of transactions.

in season s, when the match quality is ε and the price is $p^s(\varepsilon)$:

$$S_v^s(\varepsilon) \equiv p^s(\varepsilon) - (u + \beta V^w), \tag{12}$$

$$S_b^s(\varepsilon) \equiv H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w. \tag{13}$$

and the value functions for the buyer and the seller in season s are, respectively:

$$V^s = \beta V^w + u + [1 - F^s(\varepsilon^s)] E^s [S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], \tag{14}$$

$$B^s = \beta B^w + [1 - F^s(\varepsilon^s)] E^s [S_b^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s]. \tag{15}$$

A seller can count on his outside option, $\beta V^w + u$ (the flow u plus the option value of selling next season) and, with probability $[1 - F^s(\varepsilon^s)]$, on the expected surplus from a transaction for sellers. A buyer counts on his outside option, βB^w (the option value of buying next season), and, with the same probability, on the expected surplus for buyers. The two Bellman equations (14) and (15) describe the incentives of buyers and sellers in any season s. They will only agree to a transaction if they obtain a positive surplus from the transaction. In particular, it shows why a seller would agree to sell in the winter season, even though the average price is higher in the summer. A positive surplus in the winter $p^w(\varepsilon) - (u + \beta V^s) > 0$, already takes into account the potential higher price in the summer and therefore the higher value of being a seller in the summer (V^s).

We now consider the case in which prices are determined by Nash bargaining. The price maximizes the Nash product:

$$\max_{p^s(\varepsilon)} [S_v^s(\varepsilon)]^\theta [S_b^s(\varepsilon)]^{1-\theta} \quad s.t. \quad S_v^s(\varepsilon), S_b^s(\varepsilon) \geqslant 0;$$

where θ denotes the bargaining power of the seller. The solution implies

$$\frac{S_v^s(\varepsilon)}{S_b^s(\varepsilon)} = \frac{\theta}{1-\theta}, \tag{16}$$

which simplifies to (see Appendix 7.3):

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + (1-\theta)\frac{u}{1-\beta}, \tag{17}$$

a weighted average of the housing value for the matched homeowner and the present discounted value of the flow u. In other words, the price guarantees the seller the proceeds from the alternative usage of the house $(\frac{u}{1-\beta})$ and a fraction θ of the social surplus generated by the transaction $\left[H^s(\varepsilon) - \frac{u}{1-\beta}\right]$.

The average price of a transaction is:

$$P^s \equiv E^s[p^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s] = (1-\theta)\frac{u}{1-\beta} + \theta E^s[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], \tag{18}$$

which is increasing in the conditional expected surplus of housing services for transactions exceeding the reservation ε^s. Since u is aseasonal, house prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal (as we show). Moreover, the extent of seasonality is increasing in θ.

Intuitively, the source of seasonality is coming from higher average match quality in a thicker market. The higher match quality generates higher utility to the buyer. This will show up as a higher price only if the seller has some bargaining power to extract a fraction of the surplus generated from the match. To see this in equations, rewrite the average housing as the sum of two terms:

$$E^s \left[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right] = H^s(\varepsilon^s) + E^s \left[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right]. \tag{19}$$

The first term, $H^j(\varepsilon^j)$, the housing value of the marginal transaction, tends to reduce the average price in the summer since $H^s(\varepsilon^s) < H^w(\varepsilon^w)$. The second term, $E^s\left[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right]$, is the expected surplus of a transaction, tends to increase the average price in the summer due to higher match-qaulity. To see this second term more clearly, observe from (3) and (4) that

$$S^s(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}(\varepsilon - \varepsilon^s), \tag{20}$$

thus

$$E^s\left[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right] = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right],$$

It tends to increase the average housing value in the summer for two reasons. First, the probability of stay is higher in the winter, $\phi^w > \phi^s$. Second, and more important, given the assumption of first-order stochastic dominance, a higher stock of vacancies $v^s > v^w$ increases the likelihood of drawing a higher match-quality $[1 - F^s(\varepsilon)] \geqslant [1 - F^w(\varepsilon)] \quad \forall\varepsilon$. This generally leads to a higher conditional surplus in the hot season: $E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right] \geqslant E^w\left[\varepsilon - \varepsilon^w \mid \varepsilon \geqslant \varepsilon^w\right]$.[27] To sum up, due

[27] **To see this, rewrite the conditional surplus using integration by parts:**

$$E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right] = \frac{\int_{\varepsilon^s}(1 - F^s(\varepsilon))\,d\varepsilon}{1 - F^s(\varepsilon^s)}. \tag{21}$$

Putting aside the issue of the equilibrium cutoffs ε^s and ε^w (which are are quantitatively close), it follows from equation (21) that the conditional surplus is higher in the hot season, $E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right] \geqslant E^w\left[\varepsilon - \varepsilon^w \mid \varepsilon \geqslant \varepsilon^w\right]$, unless the increase in likelihood of drawing one particular level of match quality ε dominates the sum of the increase in likelihood of drawing all match quality higher than ε, i.e. unless $\frac{1-F^s(\varepsilon)}{1-F^w(\varepsilon)} > \frac{\int_\varepsilon(1-F^s(\varepsilon))d\varepsilon}{\int_\varepsilon(1-F^w(\varepsilon))d\varepsilon}$. **We cannot rule out this possibility in general, but this case does not arise in our calibration exercise. More formally, we could impose a "uniform" stochastic ordering (see Keilson and Sumita, 1982) as a sufficient condition to rule out this case. But as said such assumption is not necessary for obtaining a higher prices in the hot season.**

to both $\phi^w > \phi^s$ and the FOSD, the second term in (19) tends to increase the average price in the summer.

Given that θ affects P^s only through the equilibrium mass of vacancies (recall the reservation quality ε^s is independent of θ), it follows that the extent of seasonality in prices is increasing in θ. Since (18) holds independently of the steady state equation for v^s and v^w, this result is independent of what drives $v^s > v^w$. Note also, that the extent of seasonality in prices is decreasing in the size of the (aseasonal) flow u.

4.2.1 Comparison to Standard Asset Pricing Approach

It is useful to compare the price mechanism in our setup with that in a standard asset pricing approach. Equation (14) can be compared to the no-arbitrage condition in asset pricing. Substituting the expression for the surplus into (14), we obtain

$$V^s = [1 - F^s(\varepsilon^s)] P^s + F^s(\varepsilon^s)(\beta V^w + u)$$

The equation expresses the value of a seller as a weighted average of the market price P^s and the continuation value $(\beta V^w + u)$, with the weights given, correspondingly, by the probabilities that the transaction goes through or not. Without the search friction, a buyer will always purchase the house at the market price P^s, thus the probability of a transaction is one. In that case, the value for being a seller is $V^s = P^s$. Moreover, the surplus of a transaction is zero in a competitive equilibrium (with perfect arbitrage), so the Bellman equation (14) is equivalent to

$$P^s = \beta P^w + u = \beta(\beta P^s + u) + u \Longrightarrow P^s = \frac{u}{1-\beta},$$

and $P^s = P^w$. In other words, without the search friction, seasonality in moving probabilities ϕ^s will not be transmitted into seasonality in prices.[28]

Our price index P^j, $j = s, w$ is the average price of transactions in season j. The seasonality in price indexes, $P^s > P^w$, is due to the thick market effect, whereby matches are more likely to be

[28]Notice that with the search friction, $P^s \neq \frac{u}{1-\beta}$. From

$$V^s = \beta V^w + u + [1 - F^s(\varepsilon^s)] E^s[S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s]$$

substitute the expression for V^w and obtain:

$$V^s = \frac{u}{1-\beta} + \frac{[1 - F^s(\varepsilon^s)] E^s[S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s] + \beta[1 - F^w(\varepsilon^w)] E^w[S_v^w(\varepsilon) \mid \varepsilon \geqslant \varepsilon^w]}{1-\beta^2}$$

where the expected surpluses are strictly positive.

better in the hot season (with a higher stock of vacant houses). In what follows we focus on discussing the mechanism from the seller's perspective (a similar argument can be put forward from a buyer's perspective). The price index P^j is not the price that every seller receives. More specifically, consider a seller in the cold season who is meeting with a buyer that has a match-specific quality equal to ε. He has to decide whether to sell now at an agreed price or to wait until the hot season where the average price, P^s, is higher. Notice that the seller is not comparing P^w to P^s in his decision because what is relevant for him is not the average price P^w but rather $p^w(\varepsilon)$, which is determined between him and the buyer with quality match ε. In fact, the equilibrium value functions (14) and (15) ensure that a transaction will take place as long as the surplus is positive. More important, the option of being able to sell at a higher price in the hot season has already been incorporated into the equilibrium surpluses (12) and (13), which in turn pin down the equilibrium price $p^w(\varepsilon)$ as in (17). So even though the price of a transaction for a specific ε might be higher in the hot season, it does not follow that a seller will only transact in the hot season because of the stochastic nature of ε. By not transacting at $p^w(\varepsilon)$, a seller may end up with an even lower $p^s(\tilde{\varepsilon})$ in the hot season if he meets a buyer with a lower match quality $\tilde{\varepsilon}$, or no transaction at all if the match quality $\tilde{\varepsilon}$ is too low. So the corresponding arbitrage condition for the seller to decide whether to wait until the hot season has to consider both the probability of transacting in the hot season and the distribution of the match quality conditional on transacting. In contrast, in a standard asset-pricing model with deterministic seasons, a seller can always transact (with certainty) at market prices. The choice of whether to sell in the current season or in the next depends exclusively on the flow of benefits (or costs) of owning the house for one season relative to the expected seasonal appreciation.

4.3 Quantitative Results

4.3.1 Parameter values

We now calibrate the model to study its quantitative implications. We assume the distribution of match-quality $F(\varepsilon, v)$ follows a uniform distribution on $[0, v]$. When $v^s > v^w$ (which will follow from $\phi^w > \phi^s$), this implies first-order stochastic ordering, $F^s(.) \leqslant F^w(.)$.

We set the discount factor β so that the implied annual real interest rate is 6 percent.

We calibrate the average probability of staying in the house, $\phi = (\phi^s + \phi^w)/2$, to match survey data on the average duration of stay in a given house, which in the model is given by $\frac{1}{1-\phi}$. The median duration in the US from 1993 through 2005, according to the American Housing Survey, was 18 semesters; the median duration in the UK during this period, according to the Survey of English Housing was 26 semesters. The implied (average) moving probabilities $(1-\phi)$ per semester are hence 0.056 and 0.038 for the US and the UK, respectively. Because there is no direct data on the ex-ante ratio of moving probabilities between seasons, $(1-\phi^s)/(1-\phi^w)$, we use a range of $(1-\phi^s)/(1-\phi^w)$ from 1.1 to 1.5.[29] This implies a difference in staying probabilities between seasons, $\phi^w-\phi^s$, ranging from 0.004 to 0.015 in the UK and 0.005 to 0.022 in the US. One way to pin down the level of $(1-\phi^s)/(1-\phi^w)$ is to use data on vacancy seasonality, which is available for the US from the US Census Bureau (for the UK, data on vacancies only exist at yearly frequency). Seasonality in vacancies in the US was 31 percent during $1991-2007$.[30] As will become clear from the results displayed below, the ratio that exactly matches seasonality in US vacancies is $(1-\phi^s)/(1-\phi^w)=1.28$. The reader may want to view this as a deep parameter and potentially use it also for the UK, under the assumption that the extent of seasonality in ex-ante moving probabilities does not vary across countries.

We calibrate the flow value u to match the implied average rent-to-price ratio received by the seller. In the UK, the average gross rent-to-price ratio is roughly around 5 percent per year, according to Global Property Guide.[31] For the US, Davis et al. (2008) argue that the ratio was around 5 percent prior to 1995 when it started falling, reaching 3.5 percent by 2005. In our model, the u/P ratio (where P stands for the average price, absent seasonality) corresponds to the net rental flow received by the seller after paying taxes and other relevant costs; it is accordingly lower than the gross rent-to-price ratio. As a benchmark, we choose u so that the net rent-to-price ratio is equal to 3 percent per year (or 1.5 percent per semester), equivalent to assuming a 40 percent income tax on rent).[32] To obtain a calibrated model of u, which, as we said, is aseasonal in the data, we use the

[29] The two surveys mentioned also report the main reasons for moving. Around 30 percent of the respondents report that living closer to work or to their children's school and getting married are the main reasons for moving. These factors are of course not entirely exogenous, but they can carry a considerably exogenous component; in particular, the school calendar is certainly exogenous to housing market movements (see Goodman, 1993, and Tucker, Long, and Marx , 1995 on seasonal mobility). In all, the survey evidence supports our working hypothesis that the *ex ante* probability to move is higher in the summer (or, equivalently the probability to stay is higher in the winter).

[30] As a measure of seasonality we use, as before, the difference in annualized growth rates in vacancies between broadly defined summers and winters. The difference is statistically significant at standard levels. Vacancy is computed as the sum of houses for sale at the beginning of the season relative to the stock of houses.

[31] Data for the U.K. and other European countries can be found in

http://www.globalpropertyguide.com/Europe/United-Kingdom/price-rent-ratio

[32] In principle, other costs can trim down the 3-percent u/P ratio, including maintenance costs, and inefficiencies in the rental market that lead to a higher wedge between what the tenant pays and what the landlord receives; also,

equilibrium equations in the model without seasonality, that is, the model in which $\phi^s = \phi^w = \phi$. From (18) and (5), the average price and the reservation quality ε^d in the absence of seasonality are (see Appendix 7.3.2):

$$P = \frac{u}{1-\beta} + \theta \frac{\left[1 - \beta F\left(\varepsilon^d\right)\right] E\left[\varepsilon - \varepsilon^d \mid \varepsilon \geqslant \varepsilon^d\right]}{(1-\beta)(1-\beta\phi)}, \tag{22}$$

and

$$\frac{\varepsilon^d}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\int_{\varepsilon^d} \varepsilon dF(\varepsilon)}{1-\beta\phi F\left(\varepsilon^d\right)}. \tag{23}$$

We hence substitute $u = 0.015 \cdot P$ in the aseasonal model (equivalent to an annual rent-to-price ratio of 3 percent) for $\theta = 1/2$ (when sellers and buyers have the same bargaining power) and find the equilibrium value of P given the calibrated values for β and $F(.)$. We then use the implied value of $u = 0.015 \cdot P$ as a parameter.[33]

Finally, in reporting the results for prices, we vary θ, the seller's bargaining power parameter from 0 to 1.

4.3.2 The Extent of Seasonality

Given the calibrated values of u, β, and ϕ discussed above, Table 8 displays the extent of seasonality in vacancies and transactions generated by the model for different values of the ratio of moving probabilities (recall that seasonality in vacancies and transactions is independent of the bargaining power of the seller, θ). As throughout the paper, our metric for seasonality is the annualized difference in growth rates between the two seasons. Column (1) shows the ratio of moving probabilities, $\frac{1-\phi^s}{1-\phi^w}$. Columns (2) and (5) show the implied difference in moving probabilities between the two seasons for the US and the UK, $[(1-\phi^s) - (1-\phi^w)]$. (Recall that, because the average stay in a house differs across the two countries, a given ratio can imply different values for $\phi^w - \phi^s$, as the average probability of stay ϕ differs.) Columns (3) and (4) show the extent of seasonality in vacancies and transactions for an average stay of 9 years (as in the US) and Columns (6) and (7) show the corresponding figures for an average stay of 13 years (as in the UK)

it might not be possible to rent the house immediately, leading to lower average flows u. Note that lower values of u/p lead to even higher seasonality in prices and transactions for any given level of seasonality in moving shocks.

[33] We also calibrated the model using different values of u for different θ (instead of setting $\theta = 1/2$), keeping the ratio u/P constant. Results are not significantly different under this procedure, but the comparability of results for different values of θ becomes less clear, since u is not kept fixed.

Table 8. Seasonality in vacancies and transactions for different $\frac{1-\phi^s}{1-\phi^w}$.

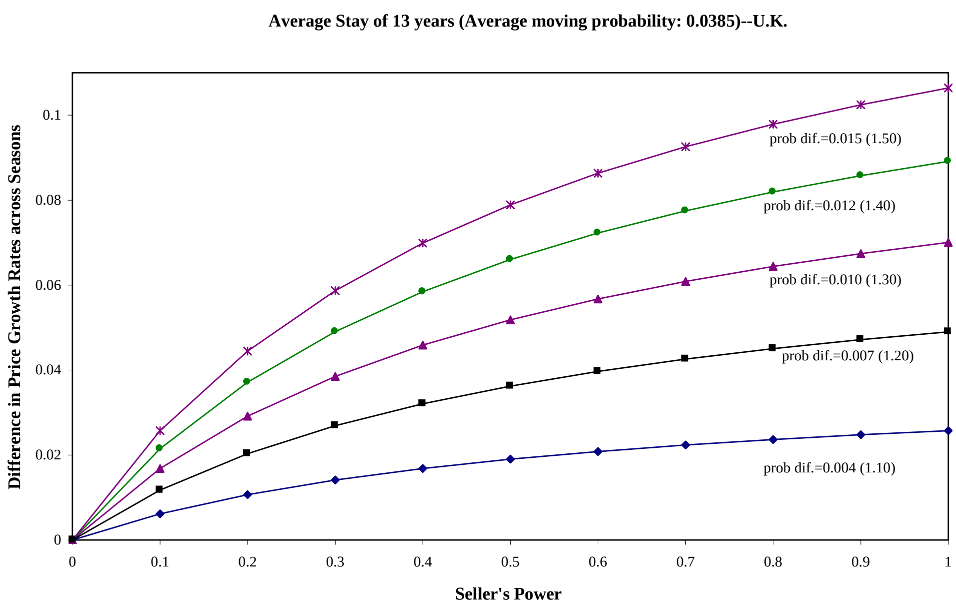
Ratio of moving probabilities between seasons (1)	***Average moving probability: 0.0556 Stay of 9 years (U.S.)***			***Average moving probability: 0.0385 Stay of 13 years (U.K.)***		
	Implied seasonal *difference* in moving probabilities (2)	Vacancies (3)	Transactions (4)	Implied seasonal *difference* in moving probabilities (5)	Vacancies (6)	Transactions (7)
1.10	0.005	12%	49%	0.004	11%	48%
1.20	0.010	23%	94%	0.007	21%	93%
1.30	0.014	33%	136%	0.010	30%	133%
1.40	0.019	42%	174%	0.013	38%	171%
1.50	0.022	51%	211%	0.015	45%	207%

The first point to note is the large amplification mechanism present in the model: For any given level of seasonality in vacancies, seasonality in transactions is at least four times bigger. Second, the Table shows that a small absolute difference in the probability to stay between the two seasons can induce large seasonality in transactions. Third, if we constrain ourselves to $\frac{1-\phi^s}{1-\phi^w} = 1.28$ to match the data on vacancies for the US, this implies a level of seasonality in transactions of about 135 percent in the US, very close to the actual 148 percent observed in the data. For the UK, ideally we would like to recalibrate the ratio $\frac{1-\phi^s}{1-\phi^w}$ to match its seasonality in vacancies; however, as said, the data are only available at yearly frequency. Using the same ratio $\frac{1-\phi^s}{1-\phi^w} = 1.28$ as a parameter for the UK would yield a seasonality in vacancies of 29 percent (the difference with the US is due to the longer duration of stay in the UK). This in turn would imply a degree of seasonality in transactions of 131 percent, somewhat above the 108 percent in the data. Note that, for a given ratio $\frac{1-\phi^s}{1-\phi^w}$, the model generates more seasonality in transactions in the US than in the UK (as in the data) because a given ratio implies a higher difference in moving probabilities $[(1-\phi^s)-(1-\phi^w)]$ in the US than in the UK, as the average stay is shorter in the former.

Seasonality in prices, as expressed earlier, depends crucially on the bargaining power of the seller, θ. Figure 8 plots the model-generated seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi^w}$, assuming an average stay of 13 years (as in the UK), and Figure 9 shows the corresponding plot for an average stay of 9 years (as in the US). As illustrated, seasonality increases with both θ and $\frac{1-\phi^s}{1-\phi^w}$. If, as before, we take $\frac{1-\phi^s}{1-\phi^w} = 1.28$ as given, the exercise implies that to match real-price seasonality in the UK (of about 6 percent, averaging between DCLG and Halifax), the bargaining power coefficient θ needs to be around 75 percent. The corresponding value for the US as a whole, with real-price seasonality just above 3 percent, is 25 percent. For US cities, as noted in Table 6, seasonality is comparable to that in the UK (with an average of 5.7 percent for real prices, using the Case-Shiller composite of cities); the model accordingly suggests that in US cities the bargaining power of sellers is considerably higher than in the economy as a whole.

The question is of course whether large differences in the bargaining power of sellers across the two countries as a whole (and between US cities and the rest of the US) are tenable. There are at least two reasons why we think this is a reasonable characterization. First, population density in the UK (246 inhabitants per km^2) is 700 percent higher than in the US (31 inhabitants per km^2), making land significantly scarcer relative to population in the UK, and potentially conferring home owners more power in price negotiations (this should also be true in denser US cities). Second, anecdotal evidence suggests that land use regulations are particularly stringent in the UK.[34] Indeed in its international comparison of housing markets, the OECD Economic Outlook 2005 highlights the "complex and inefficient local zoning regulations and slow authorization process" in the UK economy, which the report cites as one of the reasons for the remarkable rigidity of housing supply.[35] Restrictions reinforce the market power of owners by reducing the supply of houses.

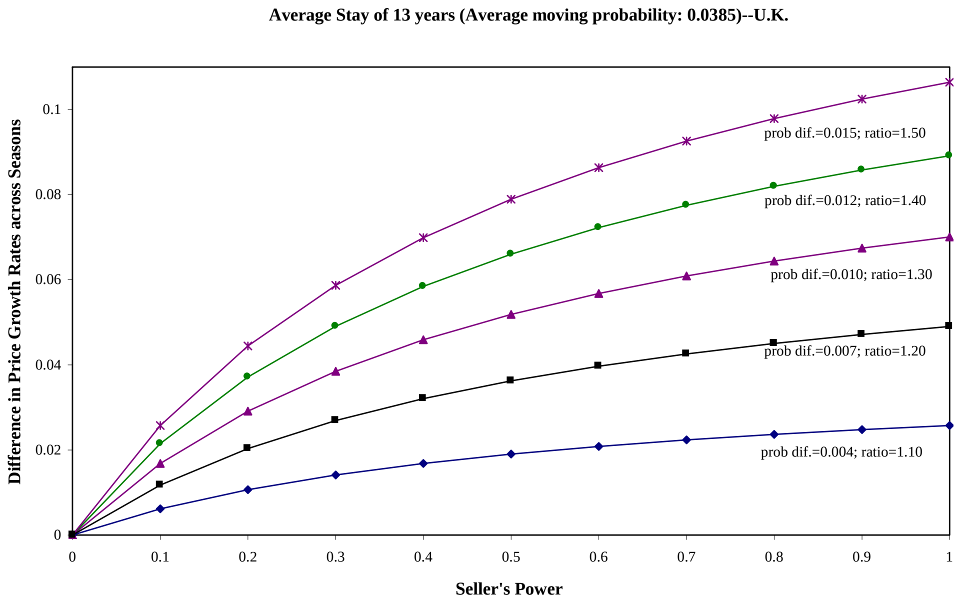
Figure 8: Seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi_w}$. UK.



[34] Again, this is likely to be true also in major cities in the US.

[35] OECD Economic Outlook 2005, Number 78, chapter III, available at http://www.oecd.org/dataoecd/41/56/35756053.pdf

Figure 9. Seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi_w}$. US.



5 Remarks on the Model

5.1 Efficiency Properties of the model

This Section discusses the efficiency of equilibrium in the decentralized economy. For a complete derivation, see Appendix 7.4. The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The key difference between the planner's solution and the decentralized solution is that the former internalizes the thick-market effect. It is evident that the equilibrium level of transactions in the decentralized economy is not socially efficient because the optimal decision rules of buyers and sellers takes the stock of vacancies in each period as given, thereby ignoring the effects of their decisions on the stock of vacant houses in the following periods. The thick-market effect generates a negative externality that makes the number of transactions in the decentralized economy inefficiently high for any given stock of vacant houses (transacting agents do not take into account that, by waiting, they can thicken the market in the following period and hence increase the overall quality of matches).[36]

The efficient level of seasonality in housing markets, however, will depend on the exact distribution of match quality $F(\varepsilon, v)$. Under likely scenarios, the solution of the planner will involve

[36]This result is similar to that in the stochastic job matching model of Pissarides (2000, chapter 8), where the reservation productivity is too low compared to the efficient outcome in the presence of search externalities.

a positive level of seasonality; that is, seasonality can be an efficient outcome. Indeed, in some circumstances, a planner may be willing to completely shut down the market in the cold season, to fully seize the benefits of a thick market.[37] This outcome is not as unlikely as one may a priori think. For example, the academic market for junior economists is extremely seasonal.[38] Extreme seasonality of course relies on the specification of utility—here we simply assume linear preferences; if agents have sufficiently concave utility functions (and intertemporal substitution across seasons is extremely low), then the planner may want to smooth seasonal fluctuations. For housing services, however, the concern of smoothing consumption across two seasons in principle should not be too strong relative to the benefit of having a better match that is on average long lasting (9 to 13 years in the two countries we analyze).

5.2 Model Assumptions

It is of interest to discuss four assumptions in the model. First, we assume that each buyer only visits one house and each seller meets only one buyer in a given season. We do this for simplicity so that we can focus on the comparison across seasons. One concern is whether allowing the buyer to visit other houses may alter the results.[39] This is, however, not the case here. Note first that the seller's outside option is also to sell to another buyer. More formally, the surplus to the buyer if the transaction for her first house goes through is:

$$\tilde{S}_b^s(\varepsilon) \equiv H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \{E^s[S_b^s(\eta)] + \beta B^w\}, \tag{24}$$

where $E^s[S_b^s(\eta)]$ is the equilibrium expected surplus (as defined in (13)) for the buyer if she goes for another house with random quality η. By definition $S_b^s(\eta) \geqslant 0$ (it equals zero when the draw for the second house η is too low). Compared to (13), the outside option for the buyer is higher because of the possibility of buying another house. Similarly, the surplus to the seller if the transaction goes through is:

$$\tilde{S}_v^s(\varepsilon) \equiv \tilde{p}^s(\varepsilon) - \{\beta V^w + u + E^s[S_v^s(\eta)]\}. \tag{25}$$

[37] The same will happen in the decentralized economy when the ratio $(1-\phi^s)/(1-\phi^w)$ is extremely high, e.g. the required ratio is larger than 10 for the calibrated parametrs we use.

[38] And it is perhaps highly efficient, given that it has been designed by our well-trained senior economists.

[39] Concretely, one might argue that the seller of the first house can now only capture part of the surplus of the buyer in excess of the buyer's second house. In this case, for the surplus (and hence prices) to be higher in the summer one would need higher dispersion of match quality in the summer. This intuition is, however, incomplete. Indeed, one can show that higher prices are obtained independently of the level of dispersion.

The key is that both buyer and seller take their outside options as given when bargaining. The price $\tilde{p}^s(\varepsilon)$ maximizes the Nash product with the surplus terms $\tilde{S}_b^s(\varepsilon)$ and $\tilde{S}_v^s(\varepsilon)$. The solution implies $(1-\theta)\tilde{S}_v^s(\varepsilon) = \theta\tilde{S}_b^s(\varepsilon)$, but the Nash bargaining for the second house implies that $(1-\theta)E^s[S_v^s(\eta)] = \theta E^s[S_b^s(\eta)]$, so:

$$(1-\theta)[\tilde{p}^s(\varepsilon) - (\beta V^w + u)] = \theta[H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \beta B^w],$$

which has the same form as (16); thus it follows that the equilibrium price equation for $\tilde{p}^s(\varepsilon)$ is identical to (17)—though the actual level of prices is different, as the cutoff match-quality is different. Our qualitative results on seasonality in prices continue to hold as before, and quantitatively they can be even stronger. Recall that in the baseline model we find that seasonality in the sum of buyer's and seller's values tends to reduce the quality of marginal transactions in the summer relative to winter because the outside option in the hot season is linked to the sum of values in the winter season: $B^w + V^w$. Intuitively, allowing the possibility of meeting another party in the same season as an outside option could mitigate this effect and hence strengthen seasonality in prices. To see this, the cutoff quality $\tilde{\varepsilon}^s$ is now defined by: $H^s(\tilde{\varepsilon}^s) = \beta(B^w + V^w) + u + E^s[S^s(\eta)]$. Compared to (4), the option of meeting another party as outside option shows up as an additional term, $E^s[S^s(\eta)]$, which is higher in the hot season.

A second simplification in the model is that buying and selling houses involve no transaction costs. This assumption is easy to dispense with. Let $\bar{\tau}_b^j$ and $\bar{\tau}_v^j$ be the transaction costs associated with the purchase $(\bar{\tau}_b^j)$ and sale $(\bar{\tau}_v^j)$ of a house in season j. Costs can be seasonal because moving costs and repairing costs may vary across seasons.[40] The previous definitions of surpluses are modified by replacing price p^j with $p^j - \bar{\tau}_v^j$ in (12) and with $p^j + \bar{\tau}_b^j$ in (13). The value functions (15) and (14), and the Nash solution (16) continue to hold as before. So, the price equation becomes:

$$p^s(\varepsilon) - \bar{\tau}_v^s = \theta[H^s(\varepsilon) - \bar{\tau}_v^s - \bar{\tau}_b^s] + (1-\theta)\frac{u}{1-\beta}, \tag{26}$$

which states that the net price received by a seller is a weighted average of housing value net of total transaction costs and the present discounted value of the flow value u. And the reservation equation becomes:

$$\varepsilon^s =: H^s(\varepsilon^s) - \left(\bar{\tau}_b^j + \bar{\tau}_v^j\right) = \beta(B^w + V^w) + u. \tag{27}$$

[40] Repair costs (both for the seller who's trying to make the house more attractive and for the buyer who wants to adapt it before moving in) may be smaller in the summer because good weather and the opportunity cost of time (assuming vacation is taken in the summer) are important inputs in construction). Moving costs, similarly, might be lower during vacation (because of both job and school holidays).

The extent of seasonality in transactions depends only on total costs ($\bar{\tau}_b^j + \bar{\tau}_v^j$) while the extent of seasonality in prices depends on the distribution of costs between buyers and sellers. One interesting result is that higher winter costs do not always result in lower winter prices. Indeed, if most of the transaction costs fall on the seller ($\bar{\tau}_v^j$ is high relative to $\bar{\tau}_b^j$), prices could actually be higher in the winter for θ sufficiently high. On the other hand, if most of the transaction costs are bared by the buyer, then seasonal transaction costs could potentially be the driver of seasonality in vacancies (and hence transactions and prices). As said, our theoretical results on seasonality in prices and transactions follow from $v^s > v^w$, independently of the particular trigger (that is, independently of whether it is seasonal transaction costs for the buyer or seasonal moving shocks; empirically, they are observationally equivalent, as they both lead to seasonality in vacancies, which we match in the quantitative exercise[41]).

Third, the model presented so far assumed observable match-quality. In Appendix 7.5 we derive the case in which the seller cannot observe the match quality ε. We model the seller's power θ in this case as the probability that the seller makes a take-it-or-leave-it offer; $1 - \theta$ is then the probability that the buyer makes a take-it-or-leave-it offer upon meeting.[42] In that setting, $\theta = 1$ corresponds to the case in which sellers always post prices. When ε is observable, a transaction goes through whenever the total surplus is positive. However, when the seller does not observe ε, a transaction goes through only when the surplus to the buyer is positive. Since the seller does not observe ε, the seller offers a price that is independent of the level of ε, which will be too high for some buyers whose $\varepsilon's$ are not sufficiently high (but whose ε would have resulted in a transaction if ε were observable to the seller). Therefore, because of the asymmetric information, the match is privately efficient only when the buyer is making a price offer. We show that our results continue to hold; the only qualitative difference is that the extent of seasonality in transactions is now decreasing in θ. This is because when ε is unobservable there is a second channel affecting a seller's surplus and hence the seasonality of reservation quality, which is opposite to the effects from the seasonality of outside option described above: When the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions; the higher the seller's power, θ, the more demanding they are and

[41] Furthermore, empirically, we are unaware of data on direct measures of moving costs or propensities to move, much less so at higher frequency.

[42] Samuelson (1984) shows that in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all.

the lower is the seasonality in transaction.

Finally, we follow the literature (see for e.g. Wheaton 1990 and Krainer 2001) by assuming exogenous moving shocks). This essentially abstracts from the decision to dissolve a match. The main potential contribution of allowing endogenous moving decision is to account for the seasonality in vacancy. Since we do not have data that is more fundamental (e.g. the seasonality in moving costs, or the seasonality in shocks that change the match quality) than the observed seasonality in vacancies, we do not attempt to predict the seasonality in vacancies. Instead, we choose to match the seasonality in vacancies observed in the data, thus the potential amplification mechanism through the endogenous moving decision is already embedded in the seasonality in vacancy. We do not dismiss the important role of endogenous moving decisions for other aspects of the housing market. We leave this for future work.

6 Concluding Remarks

This paper documents seasonal booms and busts in housing markets and argues that the predictability and high extent of seasonality in house prices cannot be quantitatively reconciled with models taking a simple asset-pricing approach.

To explain the empirical patterns, the paper presents a search-and-matching model that can quantitatively account for the seasonal fluctuations in prices and transactions observed in the US and the UK. The model sheds new light on interesting mechanisms governing fluctuations in housing markets that are likely to be present at lower frequencies. In particular, the thick-market effect that is at the core of the model's propagation mechanism does not depend on the frequency of the shocks. Lower frequency shocks associated with either business-cycle shocks or with less frequent booms and busts in housing markets could also be propagated through the same thick-market effects, to produce more amplitude in the fluctuations.

7 Appendix

7.1 Aggregate Seasonality (as Reported by the Publishers)

A first indication that house prices display seasonality comes from the observation that most publishers of house price indexes directly report SA data. Some publishers, however, report both SA and NSA data, and from these sources one can obtain a first measure of seasonality, as gauged by the publishers. For example, in the UK, Halifax publishes both NSA and SA house price series. Using these two series we computed the (logged) seasonal component of house prices as the ratio of the NSA house price series, P_t, relative to the SA series, P_t^*, from 1983:01 to 2007:04, $\left\{\ln \frac{P_t}{P_t^*}\right\}$. This seasonal component is plotted in Figure $A.1$. (Both the NSA and the SA series correspond to the UK as a whole.)

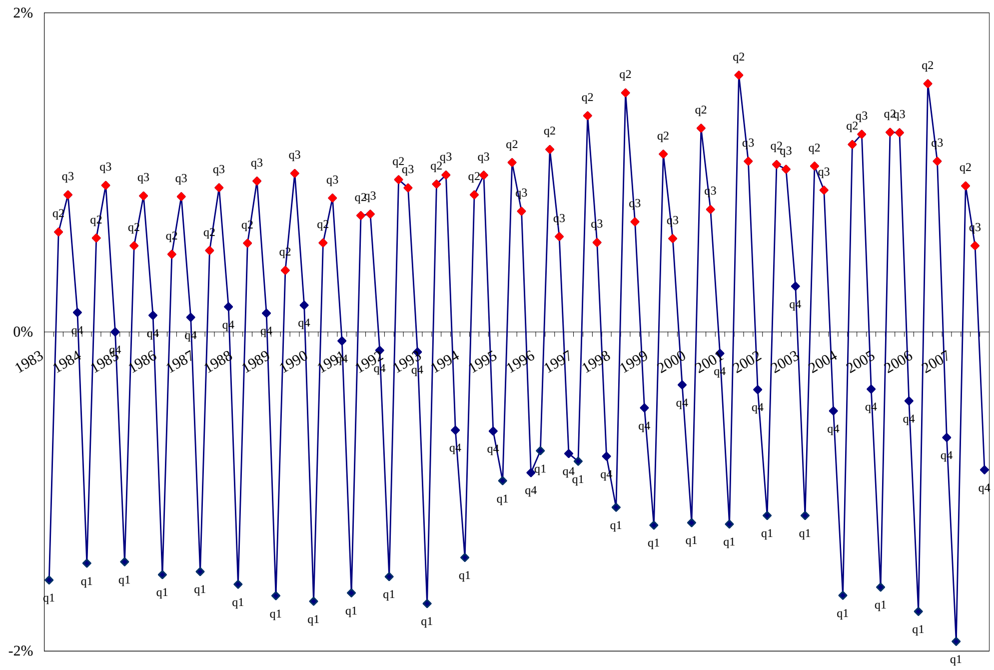
In the US, both the Office of Federal Housing Enterprise Oversight (OFHEO)'s house price index and the Case-Shiller index carried out by Standard & Poor's (S&P) are published in NSA and SA form. Figure $A.2$ depicts the seasonal component of the OFHEO series for the US as a whole, measured as before as $\left\{\ln \frac{P_t}{P_t^*}\right\}$, from 1991:01 through to 2007:04. And Figure $A.3$ shows the corresponding plot for the Case-Shiller index corresponding to a composite of 10 cities, with the data running from 1987:01 through to 2007:04. (The start of the sample in all cases is dictated by data availability.)[43]

All Figures seem to show a consistent pattern: House prices in the second and third quarters tend to rise above trend (captured by the SA series), and prices in the fourth, and particularly in the first quarter, tend to be in general at or below trend. The Figures also make it evident that the extent of price seasonality is more pronounced in the UK than in the US as a whole, though as shown in the text, certain cities in the US seem to display seasonal patterns of the same magnitude as those observed in the UK. (Some readers might be puzzled by the lack of symmetry in Figure $A.2$, as most expect the seasons to cancel out; this is exclusively due to the way OFHEO performs the seasonal adjustment;[44] for the sake of clarity and comparability across different datasets, we base our analysis only on the "raw", NSA series and hence the particular choice of seasonal adjustment by the publishers is inconsequential.)

[43] The original data in S&P's are monthly; we hence take the last month of the quarter—results are virtually identical when taking the average over the quarter.

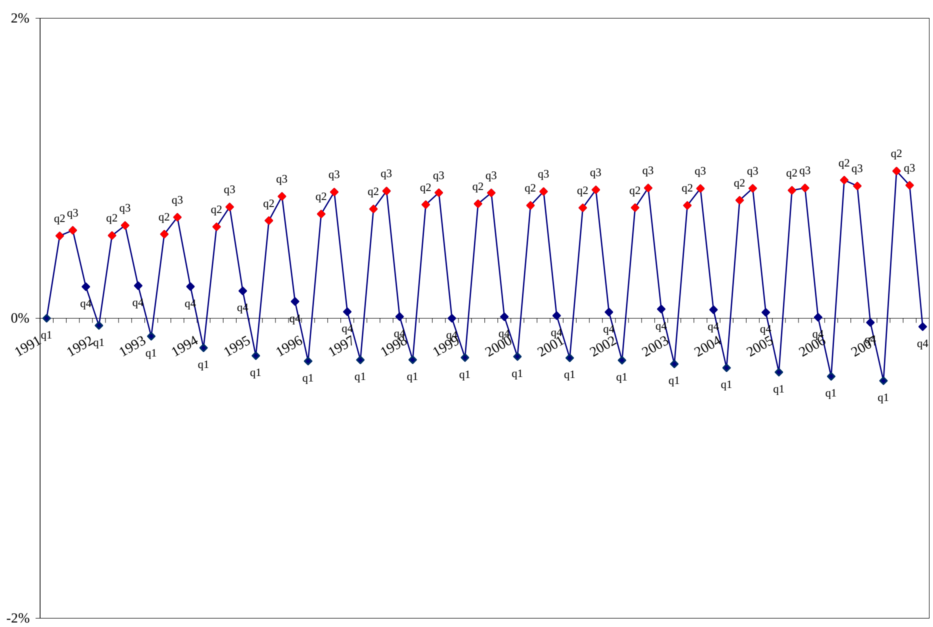
[44] OFHEO uses the Census Bureau's X-12 ARIMA procedure for SA; it is not clear, however, what the exact seasonality structure chosen is.

Figure $A.1$: Seasonal Component of House Prices in the UK 1983-2007.



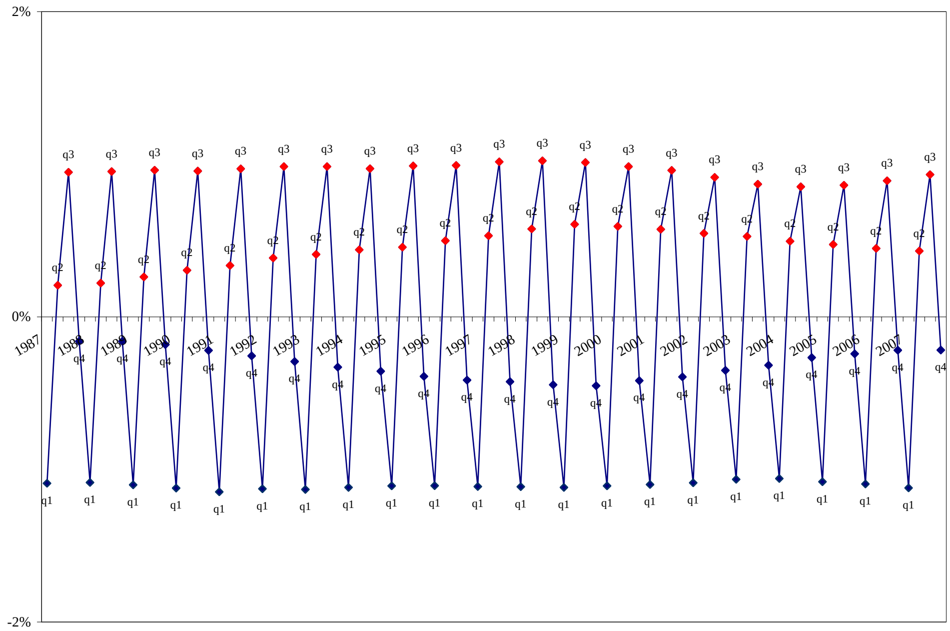
Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index. Source: Halifax.

Figure $A.2$: Seasonal Component of House Prices in the US. 1991-2007.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$; P_t is the NSA and P_t^* the SA index. Source: OFHEO.

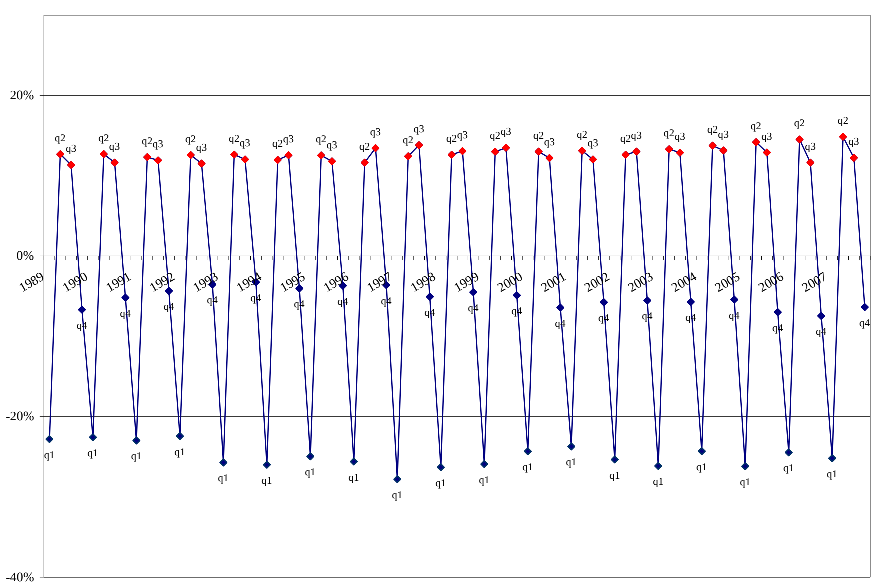
Figure $A.3$: Seasonal Component of House Prices in US cities 1987-2007.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index. Source: Case-Shiller 10-city composite.

Last, but not least, the US National Association of Realtors (NAR) publishes data on transactions both with and without SA. Figure $A.4$ plots the seasonal component of house transactions, measured (as before) as the (logged) ratio of the (NSA) number of transactions Q_t, divided by the SA number of transactions Q_t^*:$\left\{\ln \frac{Q_t}{Q_t^*}\right\}$.

Figure $A.4$: Seasonal Component of Housing Transactions in the US. 1989-2007.



Note: The plot shows $\left\{\ln \frac{Q_t}{Q_t^*}\right\}$; Q_t is the NSA and Q_t^* the SA number of transactions. Source: NAR.

The seasonal pattern for transactions is similar to that for prices: Transactions surge in the second and third quarters and stagnate or fall in the fourth and first quarters. (In the UK only NSA data for transactions are available from the publishers.)

7.2 A back-of-the-envelope calculation

We argued before that the predictability and size of the seasonal variation in housing prices pose a challenge to models of the housing market relying on standard asset-market equilibrium conditions. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal cost. Following Poterba (1984) the asset-market equilibrium conditions for any seasons $j = s$ (summer), w (winter) at time t is:[45]

$$d_{t+1,j'} + (p_{t+1,j'} - p_{t,j}) = c_{t,j} \cdot p_{t,j} \tag{28}$$

where j' is the corresponding season at time $t+1$, $p_{t,j}$ and $d_{t,j}$ are the real asset price and rental price of housing services, respectively; $c_{t,j} \cdot p_{t,j}$ is the real gross (gross of capital gains) $t-$period cost of housing services of a house with real price $p_{t,j}$; and $c_{t,j}$ is the sum of after-tax depreciation, repair costs, property taxes, mortgage interest payments, and the opportunity cost of housing equity. Note that the formula assumes away risk (and hence no expectation terms are included); this is appropriate in this context because we are focusing on a "predictable" variation of prices.[46] As in Poterba (1984), we make the following simplifying assumptions so that service-cost rates are a fixed proportion of the property price, though still potentially different across seasons ($c_{t,j} = c_{t+2,j} = c_j$, $j = s, w$): 1) Depreciation takes place at rate δ_j, $j = s, w$, constant for a given season, and the house requires maintenance and repair expenditures equal to a fraction κ_j, $j = s, w$, also constant for a given season. 2) The income-tax-adjusted real interest rate and the marginal property tax rates (for given real property prices) are constant over time, though also potentially different across seasons; they are denoted, respectively as r_j and τ_j, $j = s, w$ (in the data, as seen, they are actually constant across seasons; we come back to this point below).[47] This yields $c_j = \delta_j + \kappa_j + r_j + \tau_j$, for $j = s, w$.

[45] See also Mankiw and Weil (1989) and Muellbauer and Murphy (1997), among others.

[46] Note that Poterba's formula also implicitly assumes linear preferences and hence perfect intertemporal substitution. This is a good assumption in the context of seasonality, given that substitution across semesters (or relatively short periods of time) should in principle be quite high.

[47] We implicitly assume the property-price brackets for given marginal rates are adjusted by inflation rate, though strictly this is not the case (Poterba, 1984): inflation can effectively reduce the cost of homeownership. This, however, should not alter the conclusions concerning seasonal patterns emphasized here. As in Poterba (1984) we also assume

Subtracting (28) from the corresponding expression in the following season and using the condition that there is no seasonality in rents ($d_w \approx d_s$), we obtain:

$$\frac{p_{t+1,s} - p_{t,w}}{p_{t,w}} - \frac{p_{t,w} - p_{t-1,s}}{p_{t-1,s}} \frac{p_{t-1,s}}{p_{t,w}} = c_w - c_s \cdot \frac{p_{t-1,s}}{p_{t,w}} \tag{29}$$

Using DCLG-based results, real differences in house price growth rates for the whole of the UK are $\frac{p_s - p_w}{p_w} \simeq 8.25\%$, $\frac{p_w - p_s}{p_s} \simeq 1.06\%$,[48] the left-hand side of (29) equals $7.2\% \approx 8.25\% - 1.06\% \cdot \frac{1}{1.0106}$. Therefore, $\frac{c_w}{c_s} = \frac{0.072}{c_s} + \frac{1}{1.0106}$. The value of c_s can be pinned-down from equation (28) with $j = s$, depending on the actual rent-to-price ratios in the economy. In Table $A.1$, we summarize the extent of seasonality in service costs $\frac{c_w}{c_s}$ implied by the asset-market equilibrium conditions, for different values of d/p (and hence different values of $c_s = \frac{d_w}{p_s} + \frac{p_w - p_s}{p_s} = \frac{d_w}{p_s} + 0.0106$).

Table $A.1$: Ratio of Winter-To-Summer Cost Rates

(annualized) d/p Ratio	Relative winter cost rates $\frac{c_w}{c_s}$
1.0%	448%
2.0%	334%
3.0%	276%
4.0%	241%
5.0%	218%
6.0%	201%

As the Table illustrates, a remarkable amount of seasonality in service costs is needed to explain the differences in housing price inflation across seasons. Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 334 and 218 percent of those in the summer. Depreciation and repair costs ($\delta_j + \kappa_j$) might be seasonal, being potentially lower during the summer.[49] But income-tax-adjusted interest rates and property taxes ($r_j + \tau_j$), two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the UK is even larger. Assuming, quite conservatively, that the a-seasonal component ($r_j + \tau_j = r + \tau$) accounts for only 50 percent of the service costs in the summer ($r + \tau = 0.5c_s$), then, the formula for relative costs $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + 0.5c_s}{\delta_s + \kappa_s + 0.5c_s}$ implies that the

that the opportunity cost of funds equals the cost of borrowing.

[48] In the empirical Section we computed growth rates using difference in logs; the numbers are very close using $\frac{p_{t+1,j'} - p_{t,j}}{p_{t,j}}$ instead. We use annualized rates as in the text; using semester rates of course leads to the same results.

[49] Good weather can help with external repairs and owners' vacation might reduce the opportunity cost of time—though for this to be true it would be key that leisure were not too valuable for the owners.

ratio of depreciation and repair costs between summers and winters is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = 2\frac{c_w}{c_s} - 1$.[50] For rent-to-price ratios in the range of 2 through 5 percent, depreciation and maintenance costs in the winter should be between 568 and 336 percent of those in the summer. (If the a-seasonal component $(r+\tau)$ accounts for 80 percent of the service costs ($r + \tau = 0.8c_s$), the corresponding values are 1571 and 989 percent). By any metric, these figures seem extremely large and suggest that a deviation from the simple asset-pricing equation is called for. Similar calculations can be performed for different regions in the US; as expressed before, though the extent of price seasonality for the US as a whole is lower than in the UK, seasonality in several US cities is comparable to that in the UK and would therefore also imply large seasonality in service costs, according to condition (28).

7.3 Derivation for the model with observable value

7.3.1 Solving for prices

To derive $p^s(\varepsilon)$ in (17), use the Nash solution (16),

$$[p^s(\varepsilon) - \beta V^w - u](1-\theta) = [H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w]\theta,$$

so

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u. \tag{30}$$

Using the value functions (14) and (15),

$$(1-\theta)V^s - \theta B^s = \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u$$

solving out explicitly,

$$(1-\theta)V^s - \theta B^s = \frac{(1-\theta)u}{1-\beta}$$

substitute into (30) to obtain (17).

7.3.2 The model without seasons

The value functions for the model without seasons are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical

[50] Call λ the aseasonal component as a fraction of the summer service cost rate: $r+\tau = \lambda c_s$, $\lambda \in (0,1)$ (and hence $\delta_s + \kappa_s = (1-\lambda)c_s$). Then: $\frac{c_w}{c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{\delta_s+\kappa_s+\lambda c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{c_s}$. Or $c_w = \delta_w + \kappa_w + \lambda c_s$. Hence: $\frac{c_w - \lambda v_s}{(1-\lambda)c_s} = \frac{\delta_w+\kappa_w}{(1-\lambda)c_s}$; that is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = \frac{c_w}{(1-\lambda)c_s} - \frac{\lambda}{1-\lambda}$, which is increasing in λ for $\frac{c_w}{c_s} > 1$.

by simply setting $\phi^s = \phi^w$. Using (20), (7) and (18) to express the average price as:

$$P^s = \frac{u}{1-\beta} + \theta \left[\frac{\beta\left(1+\beta\phi^s\right)h^w\left(\varepsilon^w\right) + \left(1-\beta^2 F^s\left(\varepsilon^s\right)\right)\left(1+\beta\phi^w\right)E\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right]}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)} \right], \tag{31}$$

Using (5),

$$\frac{\varepsilon}{1-\beta\phi} = u + \frac{\beta\phi}{1-\beta\phi}\left(1-\beta\right)\left(V+B\right)$$

and $B+V$ from (7),

$$B + V = \frac{u}{1-\beta} + \frac{1}{1-\beta^2}\left\{\frac{1-F}{1-\beta\phi}E\left[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right] + \beta\frac{1-F}{1-\beta\phi}E\left[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right]\right\}$$

which reduces to:

$$B + V = \frac{u}{1-\beta} + \frac{1-F\left(\varepsilon\right)}{\left(1-\beta\right)\left(1-\beta\phi\right)}E\left(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right).$$

It follows that

$$\varepsilon = u + \frac{\beta\phi}{1-\beta\phi}\left[1-F\left(\varepsilon\right)\right]E\left(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right),$$

and the law of motion for vacancy implies:

$$v = \frac{1-\phi}{1-\phi F\left(\varepsilon\right)}.$$

7.4 Analytical derivations of the planner's solution

The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The interesting comparison is the level of reservation quality achieved by the planner with the corresponding level in the decentralized economy. To spell out the planner's problem, we follow Pissarides (2000) and assume that in any period t the planner takes as given the expected value of the housing utility service per person in period t (before he optimizes), which we denote by q_{t-1}, as well as the beginning of period's stock of vacant houses, v_t. Thus, taking as given the initial levels q_{-1} and v_0, and the sequence $\{\phi_t\}_{t=0...}$, which alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem is to choose a sequence of $\{\varepsilon_t\}_{t=0,..}$ to maximize

$$U\left(\{\varepsilon_t, q_t, v_t\}_{t=0...}\right) \equiv \sum_{t=0}^{\infty} \beta^t \left[q_t + u v_t F\left(\varepsilon_t; v_t\right)\right] \tag{32}$$

subject to the law of motion for q_t :

$$q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right), \tag{33}$$

the law of motion for v_t (which is similar to the one in the decentralized economy):

$$v_{t+1} = v_t \phi_{t+1} F(\varepsilon_t; v_t) + 1 - \phi_{t+1}, \tag{34}$$

and the inequality constraint:

$$0 \leqslant \varepsilon_t \leqslant \bar{\varepsilon}(v_t), \tag{35}$$

where the upper bound $\bar{\varepsilon}$ can potentially be infinite.

The planner faces two types of trade-offs when deciding the optimal reservation quality ε_t: A static one and a dynamic one. The static trade-off stems from the comparison of utility values generated by occupied houses and vacant houses in period t in the objective function (32). The utility per person generated from vacant houses is the rental income per person, captured by $uv_t F(\varepsilon_t)$. The utility generated by occupied houses in period t is captured by q_t, the expected housing utility service per person conditional on the reservation value ε_t set by the planner in period t. The utility q_t, which follows the law of motion (33), is the sum of the pre-existing expected housing utility q_{t-1} that survives the moving shock and the expected housing utility from the new matches. By increasing ε_t, the expected housing value q_t decreases, while the utility generated by vacant houses increases (since $F(\varepsilon_t)$ increases). The dynamic trade-off operates through the law of motion for the stock of vacant houses in (34). By increasing ε_t (which in turn decreases q_t), the number of transactions in the current period decreases; this leads to more vacant houses in the following period, v_{t+1}, and consequently to a thicker market in the next period. We first derive the case where the inequality constraints are not binding, i.e. markets are open in both the cold and hot seasons.

The Planner's solution when the housing market is open in all seasons

Because the sequence $\{\phi_t\}_{t=0,...}$ alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem can be written recursively. Taking (q_{t-1}, v_t), and $\{\phi_t\}_{t=0,..}$ as given, and provided that the solution is interior, that is, $\varepsilon_t < v_t$, the Bellman equation for the planner is given by:

$$\begin{aligned} W(q_{t-1}, v_t, \phi_t) &= \max_{\varepsilon_t} \left[q_t + uv_t F(\varepsilon_t; v_t) + \beta W\left(q_t, v_{t+1}, \phi_{t+1}\right) \right] \\ s.t. \qquad & q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF(x; v_t), \\ v_{t+1} &= v_t \phi_{t+1} F(\varepsilon_t; v_t) + 1 - \phi_{t+1}. \end{aligned} \tag{36}$$

The first-order condition implies

$$\left(1 + \beta \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial q_t}\right) v_t \left(-\varepsilon_t f(\varepsilon_t; v_t)\right) + \left(\beta \phi_{t+1} \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}} + u\right) v_t f(\varepsilon_t; v_t) = 0,$$

which simplifies to

$$\varepsilon_t\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right)=u+\beta\phi_{t+1}\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}. \tag{37}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W\left(q_{t-1},v_t,\phi_t\right)}{\partial q_{t-1}}=\phi_t\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right) \tag{38}$$

and

$$\begin{aligned}\frac{\partial W\left(q_{t-1},v_t,\phi_t\right)}{\partial v_t} &= \left(u+\beta\phi_{t+1}\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}\right)\left(F\left(\varepsilon_t;v_t\right)-v_tT_{1t}\right)\\ &\quad+\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right)\left(\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)+v_tT_{2t}\right)\end{aligned} \tag{39}$$

where $T_{1t}\equiv\frac{\partial}{\partial v_t}\left[1-F\left(\varepsilon_t;v_t\right)\right]>0$ and $T_{2t}\equiv\frac{\partial}{\partial v_t}\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)>0$. In the periodic steady state, the first order condition (37) becomes

$$\varepsilon^j\left(1+\beta\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial q^j}\right)=u+\beta\phi^{j'}\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial v^{j'}} \tag{40}$$

The envelope condition (38) implies

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}=\phi^j\left[1+\beta\left(\phi^{j'}+\beta\phi^{j'}\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}\right)\right]$$

which yields:

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}=\frac{\phi^j\left(1+\beta\phi^{j'}\right)}{1-\beta^2\phi^j\phi^{j'}} \tag{41}$$

Substituting this last expression into (39), we obtain:

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\left(u+\beta\phi^{j'}\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial v^{j'}}\right)A^j+D^j,$$

where

$$A^j\equiv F^j\left(\varepsilon^j\right)-v^jT_1^j;\qquad D^j\equiv\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j}xdF^j\left(x\right)+v^jT_2^j\right), \tag{42}$$

Hence, we have

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\left\{u+\beta\phi^{j'}\left[\left(u+\beta\phi^j\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}\right)A^{j'}+D^{j'}\right]\right\}A^j+D^j,$$

which implies

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\frac{uA^j\left(1+\beta\phi^{j'}A^{j'}\right)+\beta\phi^{j'}D^{j'}A^j+D^j}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}. \tag{43}$$

Substituting (41) and (43) into the first-order condition (40),

$$\varepsilon^j \left(1+\beta\frac{\phi^{j'}\left(1+\beta\phi^j\right)}{1-\beta^2\phi^j\phi^{j'}}\right)=u+\beta\phi^{j'}\frac{uA^{j'}\left(1+\beta\phi^jA^j\right)+\beta\phi^jD^jA^{j'}+D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}$$

simplify to:

$$\varepsilon^j\left(\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\right)=\frac{\left(1+\beta\phi^{j'}A^{j'}\right)u+\beta^2\phi^j\phi^{j'}A^{j'}D^j+\beta\phi^{j'}D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}, \tag{44}$$

and the stock of vacant houses, v^j, $j=s,w$, satisfies (8) as in the decentralized economy.

The thick-market effect enters through two terms: $T_1^j \equiv \frac{\partial}{\partial v^j}\left[1-F^j\left(\varepsilon^j\right)\right]>0$ and $T_2^j \equiv \frac{\partial}{\partial v^j}\int_{\varepsilon^j}^{\bar{\varepsilon}^j} xdF^j\left(x\right)>0$. The first term, T_1^j, indicates that the thick-market effect shifts up the acceptance schedule $\left[1-F^j\left(\varepsilon\right)\right]$. The second term, T_2^j, indicates that the thick-market effect increases the conditional quality of transactions. The interior solution (44) is an implicit function of ε^j that depends on $\varepsilon^{j'}$, v^j, and $v^{j'}$. It is not straightforward to derive an explicit condition for $\varepsilon^j<v^j$, $j=s,w$. Abstracting from seasonality for the moment, i.e. when $\phi^s=\phi^w$, it follows immediately from (8) that the solution is interior, $\varepsilon<v$. Moreover (44) implies the planner's optimal reservation quality ε^p satisfies:

$$\frac{\varepsilon^p}{1-\beta\phi}=\frac{u+\frac{\beta\phi}{1-\beta\phi}\left(\int_{\varepsilon^p}^{\bar{\varepsilon}} xdF\left(x\right)+vT_2\right)}{1-\beta\phi F\left(\varepsilon^p\right)+\beta\phi vT_1}. \tag{45}$$

Comparing (45) with (23), the thick-market effect, captured by T_1 and T_2, generates two opposite forces. The term T_1 decreases ε^p, while the term T_2 increases ε^p in the planner's solution. Thus, the positive thick-market effect on the acceptance rate T_1 implies that the number of transactions is too low in the decentralized economy, while the positive effect on quality T_2 implies that the number of transactions is too high. Since $1-\beta\phi$ is close to zero, however, the term T_2 dominates. Therefore, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome. As discussed in the text, comparing the extent in seasonality in the decentralized equilibrium to the planner's solution depends on the exact distribution $F\left(\varepsilon,v\right)$. We next derive the case in which the Planner finds it optimal to close down the market in the cold season.

The Planner's solution when the housing market is closed in the cold season

Setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$, the Bellman equation (36) can be rewritten as:

$$\begin{aligned} W^s\left(q_{t-1}^w, v_t^s\right) &= \max_{\varepsilon_t^s} \left[\begin{array}{c} \phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + u v_t^s F_t^s(\varepsilon_t^s) \\ +\beta\left(q_{t+1}^w + u\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right]\right) \\ +\beta^2 W^s\left(q_{t+1}^w, v_{t+2}^s\right) \end{array} \right] \\ & s.t. \\ q_{t+1}^w &= \phi^w\left[\phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x)\right], \\ v_{t+2}^s &= \phi^s\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right] + 1 - \phi^s. \end{aligned} \tag{46}$$

Intuitively, "a period" for the decision of ε_t^s is equal to $2t$. The state variables for the current period are given by the vector $\left(q_{t-1}^w, v_t^s\right)$, the state variables for next period are $\left(q_{t+1}^w, v_{t+2}^s\right)$, and the control variable is ε_t^s. The first order condition is:

$$\begin{aligned} 0 &= v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s f_t^s(\varepsilon_t^s) \\ &+\beta\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s \phi^w f_t^s(\varepsilon_t^s)\right) \\ &+\beta^2\left[\frac{\partial W^s}{\partial q_{t+1}^w}\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right)\right) + \frac{\partial W^s}{\partial v_{t+2}^s}\left(\phi^s v_t^s \phi^w f_t^s(\varepsilon_t^s)\right)\right], \end{aligned}$$

which simplifies to:

$$\begin{aligned} 0 &= -\varepsilon_t^s + u + \beta\left(-\phi^w \varepsilon_t^s + u\phi^w\right) \\ &+\beta^2\left[\frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\left(-\phi^w \varepsilon_t^s\right) + \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s\phi^w\right] \end{aligned}$$

and can be written as:

$$\varepsilon_t^s\left[1 + \beta\phi^w + \beta^2\phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right] = \left(1 + \beta\phi^w\right)u + \beta^2\phi^w\phi^s \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \tag{47}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial q_{t-1}^w} = \phi^s + \beta\phi^w\phi^s + \beta^2\phi^w\phi^s \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}, \tag{48}$$

and

$$\begin{aligned} &\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\ &= \left(1 + \beta\phi^w\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) + \left(1 + \beta\phi^w\right)u\left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right] \\ &+\beta^2 \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\phi^w\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) \\ &+\beta^2 \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s\phi^w\left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right], \end{aligned}$$

where $T_{1t}^s \equiv \frac{\partial}{\partial v_t^s}\left[1-F_t^s\left(\varepsilon^s\right)\right]>0$ and $T_{2t}^s \equiv \frac{\partial}{\partial v_t^s}\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x)>0$. Rewrite the last expression as:

$$
\begin{aligned}
&\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\
= &\left(1+\beta\phi^w+\beta^2\phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x)+v_t^s T_{2t}^s\right) \\
&+\left(\left(1+\beta\phi^w\right) u+\beta^2\phi^s\phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\right)\left[F_t^s\left(\varepsilon_t^s\right)-v_t^s T_{1t}^s\right]
\end{aligned} \tag{49}
$$

In steady state, (48) and (49) become

$$
\frac{\partial W^s\left(q^w, v^s\right)}{\partial q^w}=\frac{\phi^s\left(1+\beta\phi^w\right)}{1-\beta^2\phi^w\phi^s}, \tag{50}
$$

and

$$
\begin{aligned}
&\frac{\partial W^s\left(q^w, v^s\right)}{\partial v^s}\left(1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]\right) \\
= &\left(1+\beta\phi^w+\beta^2\phi^w \frac{\phi^s\left(1+\beta\phi^w\right)}{1-\beta^2\phi^w\phi^s}\right)\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x)+v^s T_2^s\right) \\
&+\left(1+\beta\phi^w\right) u\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right].
\end{aligned} \tag{51}
$$

Substituting into the FOC (47),

$$
\begin{aligned}
&\varepsilon^s \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} \\
= &\left(1+\beta\phi^w\right) u+\beta^2\phi^w\phi^s \frac{\left(1+\beta\phi^w\right) u\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]+\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x)+v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]}
\end{aligned}
$$

which simplifies to

$$
\frac{\varepsilon^s}{1-\beta^2\phi^w\phi^s}=\frac{u+\frac{\beta^2\phi^w\phi^s}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x)+v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]}, \tag{52}
$$

which is similar to the Planner's solution with no seasons in (45), with $\beta^2\phi^w\phi^s$ replacing $\beta\phi$.

7.5 Model with unobservable match quality

Assume that the seller does not observe ε. As shown by Samuelson (1984), in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all. Hence, we adopt a simple price-setting mechanism: The seller makes a take-it-or-leave-it offer p^{jv} with probability $\theta \in [0,1]$ and the buyer makes a take-it-or-leave-it offer p^{jb} with probability $1-\theta$. ($\theta=1$ corresponds to the case in which sellers post prices.) Broadly speaking, we can interpret

θ as the "bargaining power" of the seller. The setup of the model implies that the buyer accepts any offer p^{sv} if $H^s(\varepsilon) - p^{sv} \geqslant \beta B^w$; and the seller accepts any price $p^{sb} \geqslant \beta V^w + u$. Let S_v^{si} and $S_b^{si}(\varepsilon)$ be the surplus of a transaction to the seller and the buyer when the match quality is ε and the price is p^{si}, for $i = b, v$:

$$S_v^{si} \equiv p^{si} - (u + \beta V^w), \tag{53}$$

$$S_b^{si}(\varepsilon) \equiv H^s(\varepsilon) - p^{si} - \beta B^w. \tag{54}$$

Note that the definition of S_v^{si} implies that

$$p^{sv} = S_v^{sv} + p^{sb} \tag{55}$$

i.e. the price is higher when the seller is making an offer. Since only the buyer observes ε, a transaction goes through only if $S_b^{si}(\varepsilon) \geqslant 0$, $i = b, v$, i.e. a transaction goes through only if the surplus to the buyer is non-negative regardless of who is making an offer. Given $H^s(\varepsilon)$ is increasing in ε, for any price p^{si}, $i = b, v$, a transaction goes through if $\varepsilon \geqslant \varepsilon^{si}$, where

$$H^s\left(\varepsilon^{si}\right) - p^{si} = \beta B^w. \tag{56}$$

$1 - F^s(\varepsilon^{si})$ is thus the probability that a transaction is carried out. From (2), the response of the reservation quality ε^{si} to a change in price is given by:

$$\frac{\partial \varepsilon^{si}}{\partial p^{si}} = \frac{1 - \beta^2 \phi^w \phi^s}{1 + \beta \phi^w}. \tag{57}$$

Moreover, by the definition of $S_b^{si}(\varepsilon)$ and ε^{si}, in equilibrium, the surplus to the buyer is:

$$S_b^{si}(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1 + \beta \phi^w}{1 - \beta^2 \phi^w \phi^s}\left(\varepsilon - \varepsilon^{si}\right). \tag{58}$$

7.5.1 The Seller's offer

Taking the reservation policy ε^{sv} of the buyer as given, the seller chooses a price to maximize the expected surplus value of a sale:

$$\max_p \left\{\left[1 - F^s(\varepsilon^{sv})\right]\left[p - \beta V^w - u\right]\right\}$$

The optimal price p^{sv} solves

$$\left[1 - F^s(\varepsilon^{sv})\right] - \left[p - \beta V^w - u\right] f^s(\varepsilon^{sv}) \frac{\partial \varepsilon^{sv}}{\partial p^s} = 0. \tag{59}$$

Rearranging terms we obtain:

$$\underbrace{\frac{p^{sv}-\beta V^{w}-u}{p^{sv}}}_{\text{mark-up}}=\underbrace{\left[\frac{p^{sv}f^{s}\left(\varepsilon^{sv}\right)\frac{\partial\varepsilon^{s}}{\partial p^{s}}}{1-F^{s}\left(\varepsilon^{sv}\right)}\right]^{-1}}_{\text{inverse-elasticity}},$$

which makes clear that the price-setting problem of the seller is similar to that of a monopolist who sets a markup equal to the inverse of the elasticity of demand (where demand in this case is given by the probability of a sale, $1-F^{s}\left(\varepsilon^{s}\right)$). The optimal decisions of the buyer (57) and the seller (59) together imply:

$$S_{v}^{sv}=\frac{1-F^{s}\left(\varepsilon^{sv}\right)}{f^{s}\left(\varepsilon^{sv}\right)}\frac{1+\beta\phi^{w}}{1-\beta^{2}\phi^{w}\phi^{s}}. \tag{60}$$

Equation (60) says that the surplus to a seller generated by the transaction is higher when $\frac{1-F^{s}(\varepsilon^{sv})}{f^{s}(\varepsilon^{sv})}$ is higher, i.e. when the conditional probability that a successful transaction is of match quality ε^{sv} is lower. Intuitively, the surplus of a transaction to a seller is higher when the house is transacted with a stochastically higher match quality, or loosely speaking, when the distribution of match quality has a "thicker" tail.

Given the price-setting mechanism, in equilibrium, the value of a vacant house to its seller is:

$$V^{s}=u+\beta V^{w}+\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}. \tag{61}$$

Solving out V^{s} explicitly,

$$V^{s}=\frac{u}{1-\beta}+\theta\frac{\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right]S_{v}^{wv}}{1-\beta^{2}}, \tag{62}$$

which is the sum of the present discounted value of the flow value u and the surplus terms when its seller is making the take-it-or-leave-it offer, which happens with probability θ. Using the definition of the surplus terms, the equilibrium p^{sv} is:

$$p^{sv}=\frac{u}{1-\beta}+\theta\frac{\left[1-\beta^{2}F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right]S_{v}^{wv}}{1-\beta^{2}}. \tag{63}$$

7.5.2 The Buyer's Offer

The buyer offers a price that extracts all the surplus from the seller, i.e.

$$S_{v}^{sb}=0\Leftrightarrow p^{sb}=u+\beta V^{w}$$

Using the value function V^{w} from (62), the price offered by the buyer is:

$$p^{sb}=\frac{u}{1-\beta}+\theta\frac{\beta^{2}\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right]S_{v}^{wv}}{1-\beta^{2}}. \tag{64}$$

The buyer's value function is:

$$
\begin{aligned}
B^s &= \beta B^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] \\
&\quad + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right],
\end{aligned} \tag{65}
$$

where $E^s[.]$ indicates the expectation taken with respect to the distribution $F^s(.)$. Since the seller does not observe ε, the expected surplus to the buyer is positive even when the seller is making the offer (which happens with probability θ). As said, buyers receive zero housing service flow until they find a successful match. Solving out B^s explicitly,

$$
\begin{aligned}
B^s &= \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] \\
&\quad + \beta\left\{\theta\left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S_b^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S_b^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}.
\end{aligned} \tag{66}
$$

7.5.3 Reservation quality

In any season s, the reservation quality ε^{si}, for $i = v, b$, satisfies

$$
H^s\left(\varepsilon^{si}\right) = S_v^{si} + u + V^w + \beta B^w, \tag{67}
$$

which equates the housing value of a marginal owner in season s, $H^s(\varepsilon^s)$, to the sum of the surplus generated to the seller (S_v^{si}), plus the sum of outside options for the buyer (βB^w) and the seller ($\beta V^w + u$). Using (2), ε^{si} solves:

$$
\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} = S_v^{si} + u + \frac{\beta\phi^w\left(1-\beta^2\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^w + V^w\right) - \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^s + B^s\right). \tag{68}
$$

The reservation quality ε^s depends on the sum of the outside options for buyers and sellers in both seasons, which can be derived from (62) and (66):

$$
\begin{aligned}
& B^s + V^s \\
= \; & \frac{u}{1-\beta} + \\
& \theta\left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] + \\
& \beta\left\{\theta\left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\},
\end{aligned} \tag{69}
$$

where $S^{si}(\varepsilon) \equiv S_b^{si}(\varepsilon) + S_v^{si}$ is the total surplus from a transaction with match quality ε. Note from (68) that the reservation quality is lower when the buyer is making a price offer: $\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon^{sv} - \varepsilon^{sb}\right) = S_v^{sv}$. Also, because of the asymmetric information, the match is privately efficient when the buyer is making a price offer.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $\left(p^{jv}, p^{jb}, \varepsilon^{jv}, \varepsilon^{jb}\right)$ in season $j = s, w$ through two channels, as shown in (63), (64), and(68)): the conditional density of the distribution at reservation ε^{jv}, i.e. $\frac{f^j\left(\varepsilon^{jv}\right)}{1-F^j\left(\varepsilon^{jv}\right)}$, and the expected surplus quality above reservation ε^{jv}, i.e. $\left(1 - F^j\left(\varepsilon^{ji}\right)\right) E^j\left[\varepsilon - \varepsilon^{ji} \mid \varepsilon \geqslant \varepsilon^{ji}\right]$, $i = b, v$. As shown in (60), a lower conditional probability that a transaction is of marginal quality ε^{jv} implies higher expected surplus to the seller S_v^{jv}, which increases the equilibrium prices p^{jv} and p^{jb} in (63) and (64). Similarly as shown in (58) and the assumption of first order stochastic dominance, using integration by parts, expected surplus to the buyer $\left(1 - F^j\left(\varepsilon^{ji}\right)\right) E^s\left[S_b^{si}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{si}\right]$, $i = b, v$ is higher in the hot season with higher vacancies. These two channels affect V^j and B^j in (62) and (66), and as a result affect the reservation qualities ε^{jv} and ε^{jb} in (5).

7.5.4 Stock of vacant houses

In any season s, the average probability that a transaction goes through is $\left\{\theta\left[1 - F^s\left(\varepsilon^{sv}\right)\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right]\right\}$, and the average probability that a transaction does not through is $\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}$. Hence, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$\begin{aligned} v^s &= (1-\phi^s)\left\{v^w\left[\theta\left(1 - F^w\left(\varepsilon^{wv}\right)\right) + (1-\theta)\left(1 - F^w\left(\varepsilon^{wb}\right)\right)\right] + 1 - v^w\right\} \\ &\quad + v^w\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}, \end{aligned}$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = v^w \phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\} + 1 - \phi^s, \tag{70}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality $\left(\varepsilon^{wv}, \varepsilon^{wb}\right)$ and on the distribution $F^w(.)$.

An equilibrium is a vector $\left(p^{sv}, p^{sb}, p^{wv}, p^{wb}, B^s + V^s, B^w + V^w, \varepsilon^{sv}, \varepsilon^{sb}, \varepsilon^{wv}, \varepsilon^{wb}, v^s, v^w\right)$ that jointly satisfies equations (63),(66),(68), (69) and (70), with the surpluses S_v^j and $S_b^j(\varepsilon)$ for $j = s, w$, derived as in (60), and (58). Using (70), the stock of vacant houses in season s is given by:

$$v^s = \frac{(1-\phi^w)\phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\} + 1 - \phi^s}{1 - \phi^w\phi^s\left\{\theta F^s\left(\varepsilon^{sv}\right) + (1-\theta) F^s\left(\varepsilon^{sb}\right)\right\}\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}}. \tag{71}$$

Given $1 - \phi^s > 1 - \phi^w$, as in the observable case, it follows that, in equilibrium $v^s > v^w$.

7.5.5 Seasonality in Prices

Let

$$p^s \equiv \frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]p^{sv}+(1-\theta)\,p^{sb}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

be the average price observed in season s. Given $p^{sv}=S_v^{sv}+p^{sb}$, we can rewrite it as

$$p^s = p^{sb}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

using (64)

$$\begin{aligned} p^s &= \frac{u}{1-\beta}+\theta\frac{\beta^2\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}+\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{1-\theta F^s\left(\varepsilon^{sv}\right)} \\ &= \frac{u}{1-\beta}+\theta\left(\frac{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\beta^2+1-\beta^2}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}\right)\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}+\frac{\theta\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2} \end{aligned}$$

we obtain,

$$p^s = \frac{u}{1-\beta}+\theta\left\{\frac{\left[1-\theta\beta^2F^s\left(\varepsilon^{sv}\right)\right]\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}+\frac{\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}\right\}. \tag{72}$$

Since the flow u is a-seasonal, house prices are seasonal if $\theta>0$ and the surplus to the seller is seasonal. As in the case with observable match quality, when sellers have some "market power" ($\theta>0$), prices are seasonal. The extent of seasonality is increasing in the seller's market power θ. To see this, note that the equilibrium price is the discounted sum of the flow value (u) plus a positive surplus from the sale. The surplus S_v^{sv}, as shown in (60), is seasonal. Given $v^s>v^w$, Assumption 2 implies hazard rate ordering, i.e. $\frac{f^w(x)}{1-F^w(x)}>\frac{f^s(x)}{1-F^s(x)}$ for any cutoff x, i.e. the thick-market effect lowers the conditional probability that a successful transaction is of the marginal quality ε^{sv} in the hot season, that is, it implies a "thicker" tail in quality in the hot season. In words, the quality of matches goes up in the summer and hence buyers' willingness to pay increases; sellers can then extract a higher surplus in the summer; thus, $S_v^{sv}>S_v^{wv}$. As in the case with observable ε, there is an equilibrium effect through the seasonality of cutoffs. As shown in (68), the equilibrium cutoff ε^{sv} depends on the surplus to the seller (S_v^{sv}) and on the sum of the seller's and the buyer's outside options, while the equilibrium cutoff ε^{sb} depends only on the sum of the outside options. The seasonality in outside options tends to reduce $\varepsilon^{si}/\varepsilon^{wi}$ for $i=b,v$. This is because the outside option in the hot season s is determined by the sum of values in the winter season: B^w+V^w, which is lower than in the summer. However, the seasonality in the surplus term, $S_v^{sv}>S_v^{wv}$ (shown before), tends to increase $\varepsilon^{sv}/\varepsilon^{wv}$ (the marginal house has to be of higher quality in order to generate a bigger surplus to the seller). Because of these two opposing forces, the equilibrium effect is likely to be small (even smaller than in the observable case.)

Given that θ affects S_v^{sv} only through the equilibrium vacancies and reservation qualities, it follows that the extent of seasonality in price is increasing in θ.

7.5.6 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[\theta \left(1 - F^w \left(\varepsilon^{wv}\right)\right) + (1-\theta)\left(1 - F^w\left(\varepsilon^{wb}\right)\right)\right]. \tag{73}$$

(An isomorphic expression holds for Q^w). As in the observables case, seasonality in transactions stems from three sources. First, the direct effect from a larger stock of vacancies in the summer, $v^s > v^w$. Second the amplification through the thick-market effects that shifts up the probability of a transaction. Third, there is an equilibrium effect through cutoffs. As pointed out before, this last effect is small. As in the case with observable ε, most of the amplification stems from the thick-market effect. What is new when ε is unobservable is that the extent of seasonality in transactions is decreasing in the seller's market market power θ. This is because higher θ leads to higher surplus in the summer relative to winter, S_v^{sv}/S_v^{wv}, which in turn increases $\varepsilon^{sv}/\varepsilon^{wv}$ and hence decreases Q^s/Q^w); the higher is θ, the stronger is this effect (it disappear when $\theta = 0$).

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